PULSED POWER SYSTEM
脈衝功率系統

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2023 Fall Semester

Tuesday 9:10-12:00

Lecture 1

http://capst.ncku.edu.tw/PGS/index.php/teaching/

Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=md577c3633c5970f80cbc9e821927e016

2023/9/5 updated

Grading

- **Weekly presentations – 30 %**
  - **Class review.**
- **Final presentations – 70 %**
  - **Design of a pulsed-power system – 35 %.**
  - **Applications of pulsed-power system – 35 %.**

- **No class on 10/31!!!**

Reference

- Foundations of pulsed power technology, by Jane Lehr & Pralhad Ron
- Pulsed power systems, by H. Bluhm
- Pulsed power, by Gennady A. Mesyats
- J. C. Martin on pulsed power, edited by T. H. Martin, A. H. Guenther, and M. Kristiansen
- Pulse power formulary, by Richard J. Adler

- Circuit analysis, by Cunningham and Stuller

Outlines

- Introduction to pulsed-power system
- Review of circuit analysis
- Static and dynamic breakdown strength of dielectric materials
  - Gas – Townsend discharge (avalanche breakdown), Paschen's curve
  - Liquid
  - Solid
- Energy storage
  - Pulse discharge capacitors
  - Marx generators
  - Inductive energy storage

Outlines

- **Switches**
  - **Closing switches**
  - **Opening switches**
- **Pulse-forming lines**
  - **Blumlein line**
  - **Pulse-forming network**
  - **Pulse compressor**
- **Pulse transmission and transformation**
  - **Self-magnetic insulation**
  - **Pulse transformer**
  - **Voltage multiplier**
  - **H-bridge pulse generator**
  - **Pulse-width modulation (PWM)**
  - **Fast high-voltage pulse generator**

Outlines

- Power and voltage adding
  - Marx generator
  - LC generator
  - Line pulse transformers
  - Induction voltage adder (IVA)
  - Linear induction accelerator (LIA)
  - Linear transformer driver (LTD)
- Diagnostics
  - Voltage measurement
  - Current measurement
- Applications of pulsed-power system

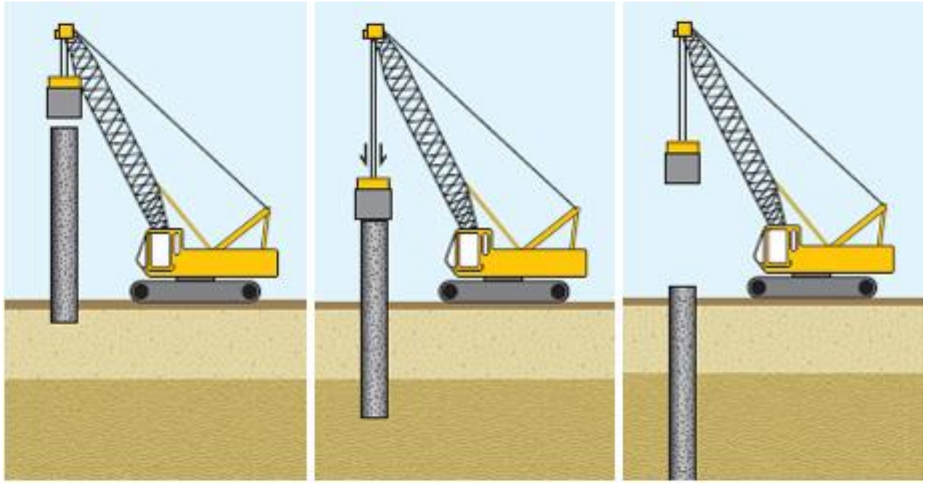
Outlines

- **Introduction to pulsed-power system**
- **Review of circuit analysis**
- **Static and dynamic breakdown strength of dielectric materials**
  - **Gas – Townsend discharge (avalanche breakdown), Paschen's curve**
  - **Liquid**
  - **Solid**
- **Energy storage**
  - **Pulse discharge capacitors**
  - **Marx generators**
  - **Inductive energy storage**

Pulsed-power system release the stored energy in a short period of time to provide high power output

- **Pulsed power is a scheme where stored energy is discharged as electrical energy into a load in a short pulse or short pulses with a controllable repetition rate.**
- **Driven piles - prefabricated steel, wood or concrete piles are driven into the ground using impact hammers**

- **Driven piles**
- **Hammer**



PLACEMENT OF PILE   INSTALLATION OF PILE   REPETITION OF PROCESS

http://www.saudifoundations.com/driven.html
http://learnhowtowritesongs.com/tag/thesaurus/

Example of short pulses with a controllable repetition rate

https://www.youtube.com/watch?v=5fe8b4MlPYw

In general, a pulsed-power system provides a power in the order of 1 GW

- The highest energy and power that have been achieved in a simgle pulse are in the order of 100 MJ & few hundreds TW, respectively.

	General cases	Our system
Energy per pulse	1 ~ 10 MJ	1 kJ
Peak power	1 MW ~ 100 TW	0.6 GW
Peak voltage	1 kV ~ 10 MV	20 kV
Peak current	1 kA ~ 10 MA	135 kA
Pulse width	0.1 ns ~ 10 us	1 us

Physiological Effects of an Electric Shock

UR LLE

Effects of Electrical Current* on the Body[3]

Current	Reaction
1 milliamp	Just a faint tingle.
5 milliamps	Slight shock felt. Disturbing, but not painful. Most people can "let go." However, strong involuntary movements can cause injuries.
6–25 milliamps (women)† **9–30 milliamps** (men)	Painful shock. Muscular control is lost. This is the range where "freezing currents" start. It may not be possible to "let go."
50–150 milliamps	Extremely painful shock, respiratory arrest (breathing stops), severe muscle contractions. Flexor muscles may cause holding on; extensor muscles may cause intense pushing away. Death is possible.
1,000–4,300 milliamps (1–4.3 amps)	Ventricular fibrillation (heart pumping action not rhythmic) occurs. Muscles contract; nerve damage occurs. Death is likely.
10,000 milliamps (10 amps)	Cardiac arrest and severe burns occur. Death is probable.
15,000 milliamps (15 amps)	Lowest overcurrent at which a typical fuse or circuit breaker opens a circuit!

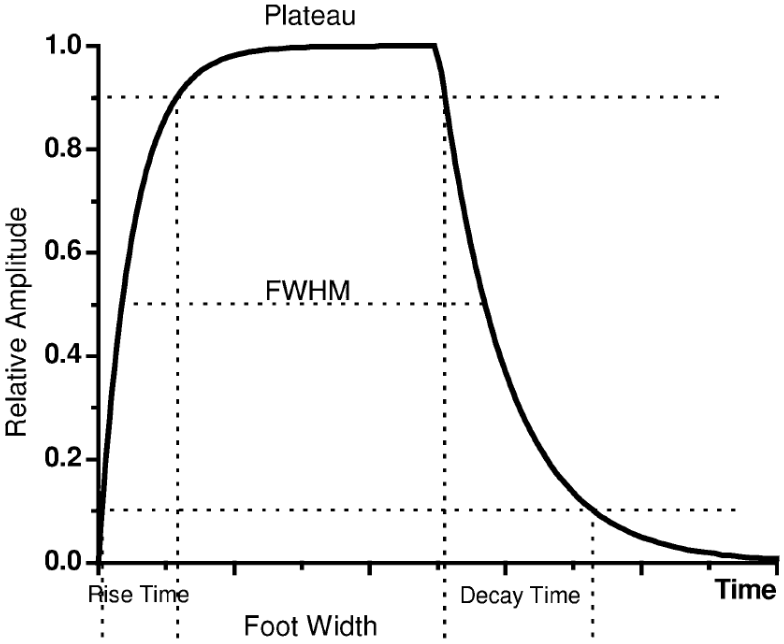
*Effects are for voltages less than about 600 volts. Higher voltages also cause severe burns.
†Differences in muscle and fat content affect the severity of shock.

Electric shock victims suffering from ventricular fibrillation will die if they do not receive prompt, emergency medical attention

A pulse is characterized by its shapes

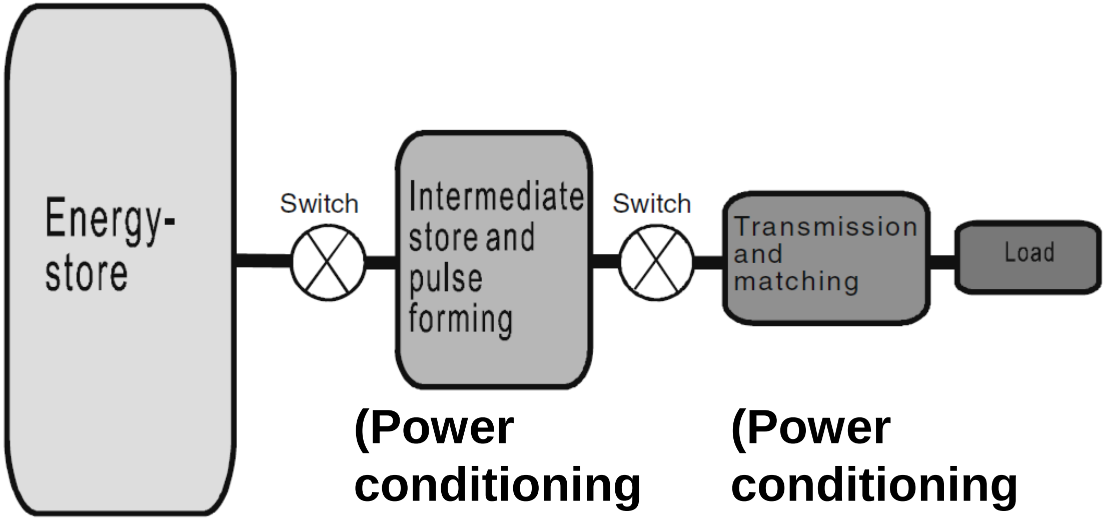
- The shape of a pulse is characterized by:
  - Rise time: from 10 % to 90 % of the plateau
  - Fall time: from 90 % to 10 % of the plateau

  (Rise & fall time depend on the evolution of the "load impedance," which in most cases varies with time.
  - Duration:
    - FWHM
    - Width of 90% of the peak amplitude
  - Flatness of the plateau region: important for some applications such as for driving a Pockel's cell.



A pulsed-power system has an energy bank that is charged slowly and store the energy for some time

- A generator scheme for the production of high-power electrical pulses is always based on an energy store that is charged slowly at a relatively low charging power and is discharged rapidly by activating a switch.
- To achieve the desired power magnification factor and to shape the pulse, the above process can be repeated several time.

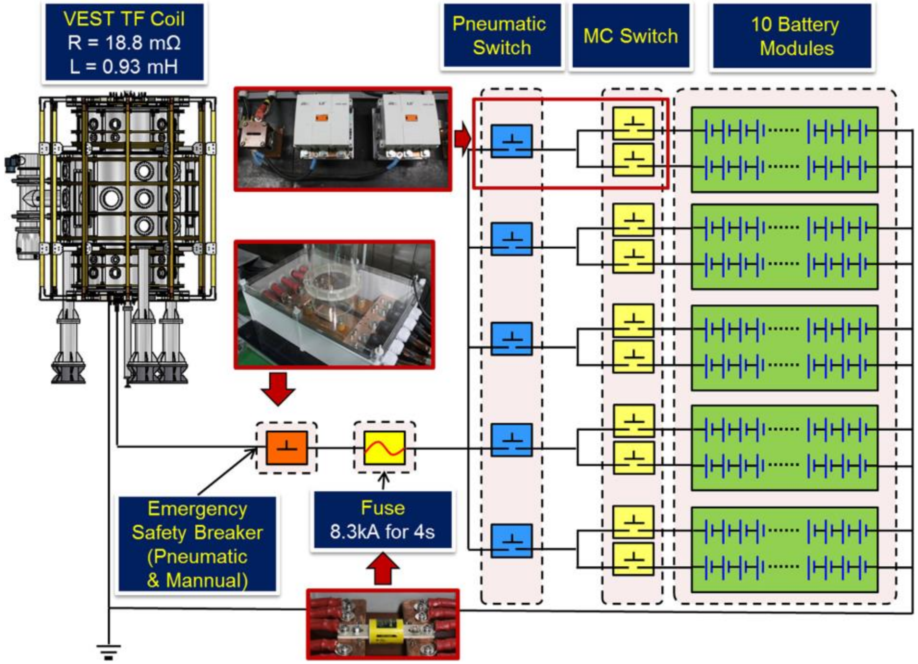


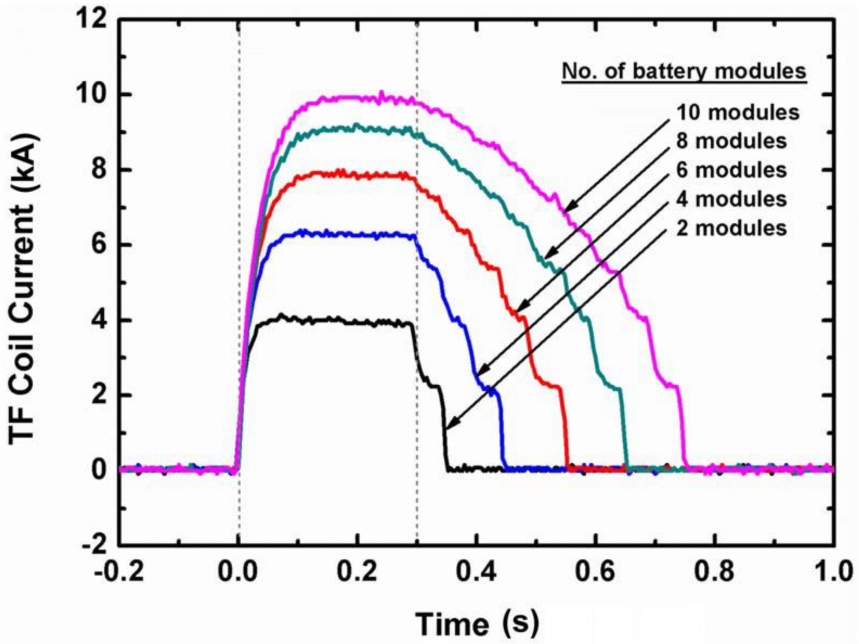
(Power conditioning stage 1)

(Power conditioning stage 2)

- The energy can be stored either chemically (battery), mechanically, or electrically.

Batteries can be used for energy storage



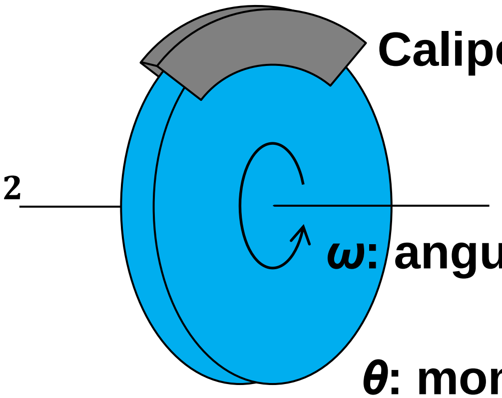


B.K. Jung, etc.,Fusion Eng. Design 88, 1597 (2013)
Chung K. J. et al, Plasma Sci. Technol. 15, 244 (2013)

A flywheel can store energy mechanically

- **Mechanical energy:** $E_{\text{kin}} = \frac{1}{2}\theta\omega^2$
- **For a massive cylinder:**

$$\theta = \frac{1}{2}Mr^2 \Rightarrow E_{\text{kin}} = \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2$$



- **Stored energy density:**

$$E_M = \frac{E_{\text{kin}}}{M} = \frac{1}{4}r^2\omega^2$$

- **The ultimate energy density is limited by the tensile strength of the material used to construct the flywheel.**

$$\Sigma \sim \rho\ {\omega_{\max}}^2\ r^2$$

- **For a AISI 302 stainless steel cylinder with a radius of 1 m:**

$$\Sigma = 860\text{MPa} \qquad \rho = 8190\,\text{kg}/m^3$$

$$\omega_{\max} \sim 300\left(\text{sec}^{-1}\right) \qquad E_M \sim 2\times 10^4\,J/\text{kg} \sim 1.6\times 10^8\,J/m^3$$

- **The problem with mechanical storage is to release the energy in a sufficiently short time. Several electrical compression stages are needed in combination with the mechanical storage to achieve the desired power level.**

Electrical energy can be stored either capacitively in an electric field or inductively in a magnetic field

- **Electric field:** $E_e = \frac{1}{2}\epsilon_r \epsilon_o E^2$
  - **Oilimpregnated paper:** $\epsilon_r = 6 \quad E_{\text{break}} = 0.78 \times 10^8\, V/m$

$$E_e = \frac{1}{2} \times 6 \times 8.85 \times 10^{-12} \times \left(0.78 \times 10^8\right)^2 \sim 160\, \text{kJ}/m^3$$

  - **With the finite packing density**

$$E_e{}' = \frac{1}{2} \times E_e \sim 8 \times 10^4\, J/m^3$$

Electrode @ V

Dielectric

Electrode @ V=0

Electrical energy can be stored either capacitively in an electric field or inductively in a magnetic field

- **Magnetic field:** $E_B = \frac{1}{2}\frac{B^2}{\mu_r \mu_o}$
  - The maximum energy density is limited by the onset of melting at the conductor surface or by the mechanical strength of the storage inductor.
  - Coil heating: $c_v \rho \mathrm{T} = \frac{1}{2\mu_0} B^2 \theta \sim \frac{B^2}{2\mu_0}$

c_v : Heat capacitor per unit mass
ρ : Mass density
T : Surface temperature
θ : A factor depending on the form of the pulse

$$B \times l = \mu_0 N I$$

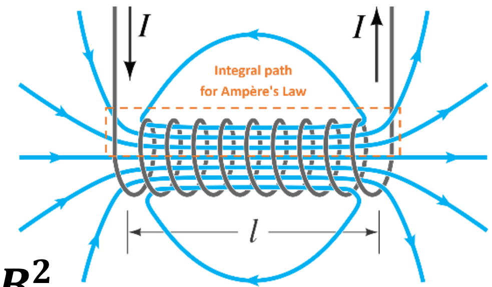
$$B = \mu_0 n I \sim I$$

$$P = I^2 R \sim B^2 \mathrm{R}$$

  - Copper:

$$\left. \begin{aligned} c_v &= 0.385\, J/g-k \\ T_{\text{melting}} &= 1085^o C \\ \rho &= 8960\, \text{kg}/m^3 \end{aligned} \right\} B \sim 100\ \mathrm{T}$$

$$\Sigma_{\text{copper}} = 70\text{MPa}$$



  - The coil needs to hold the magnetic pressure: $\frac{B^2}{2\mu_0} = P_B \leq \Sigma$

$$B \sim 13\ \mathrm{T}$$

Electrical energy can be stored either capacitively in an electric field or inductively in a magnetic field

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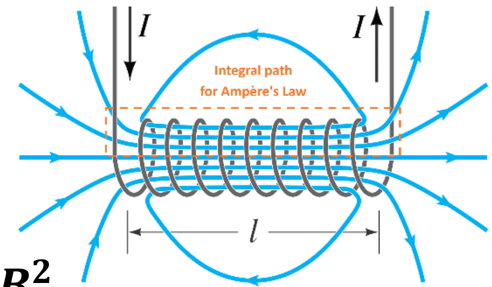


  - The coil needs to hold the magnetic pressure: $\frac{B^2}{2\mu_0} = P_B \leq \Sigma$

$B \sim 13$ T ⇨ $E_B \sim 7 \times 10^7\, J/m^3$

More energy can be stored in a magnetic field

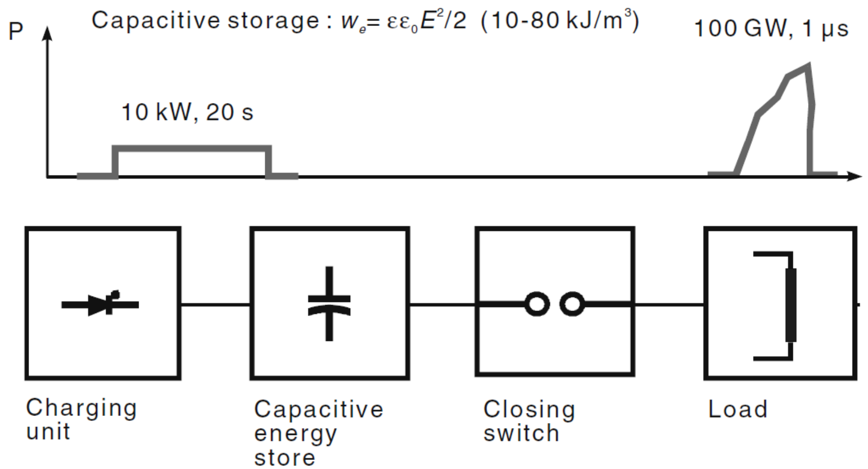
Mechanical Energy	Electrical energy	Magnetic energy
1.6×10^{8} J/m^3	8×10^{4} J/m^3	7×10^{7} J/m^3

- **The energy density stored in a magnetic field can be about 2~3 orders of magnitude higher than that storable in a electric field!**
- **Capacitive storage:**
  - **Requires one or more closing switches which remain open during charging and hold the charging voltage.**
  - **Power multiplication is done by current amplification.**
- **Inductive storage:**
  - **Requires an opening switch which is closed during charge-up, carrying a large current at this stage.**
  - **Power multiplication is done by voltage amplification.**

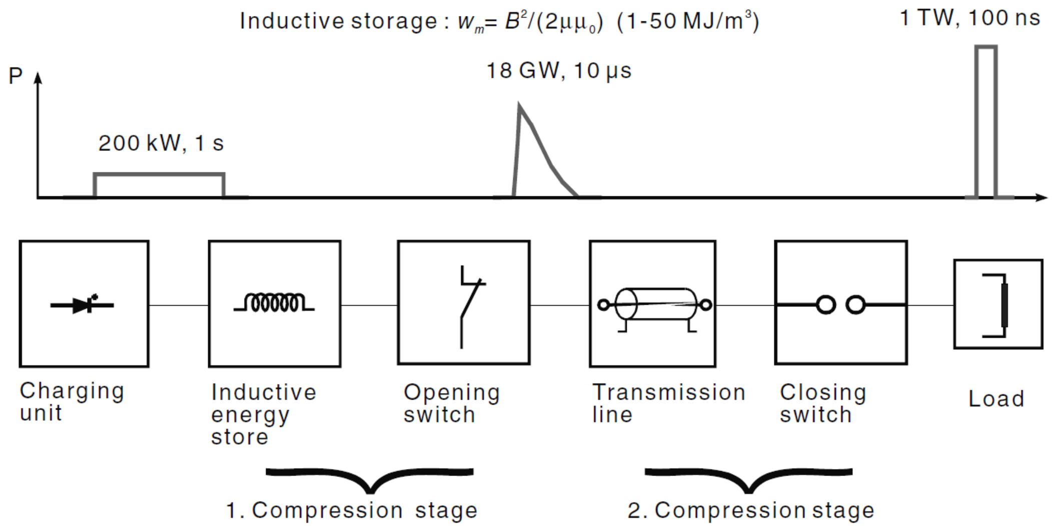
- **Opening switches are harder to operate then closing switches. They are generally slower leading to a lower power output.**

It is more complicated to use inductive storage

- **Capacitive storage:**

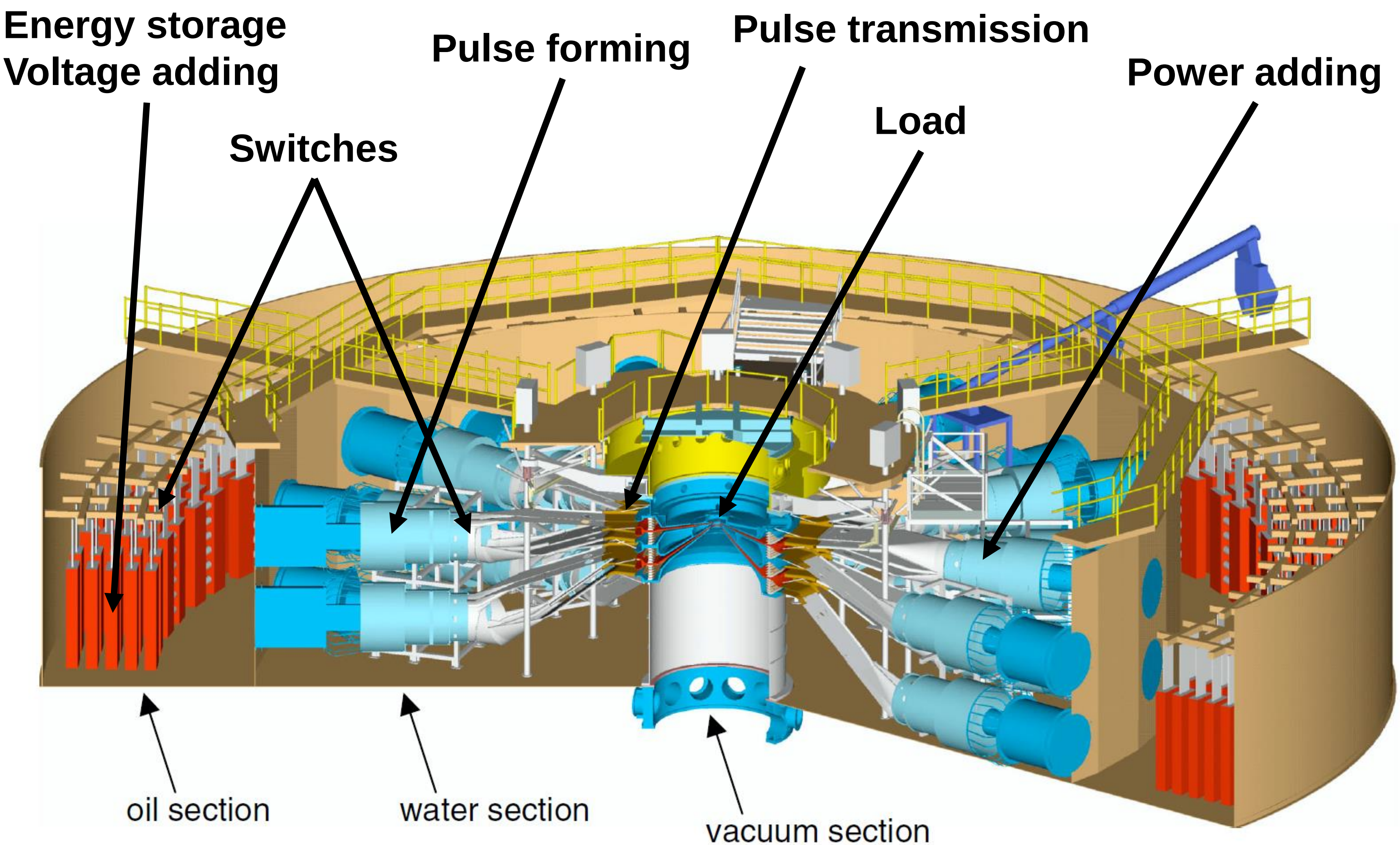


- **Inductive storage:**



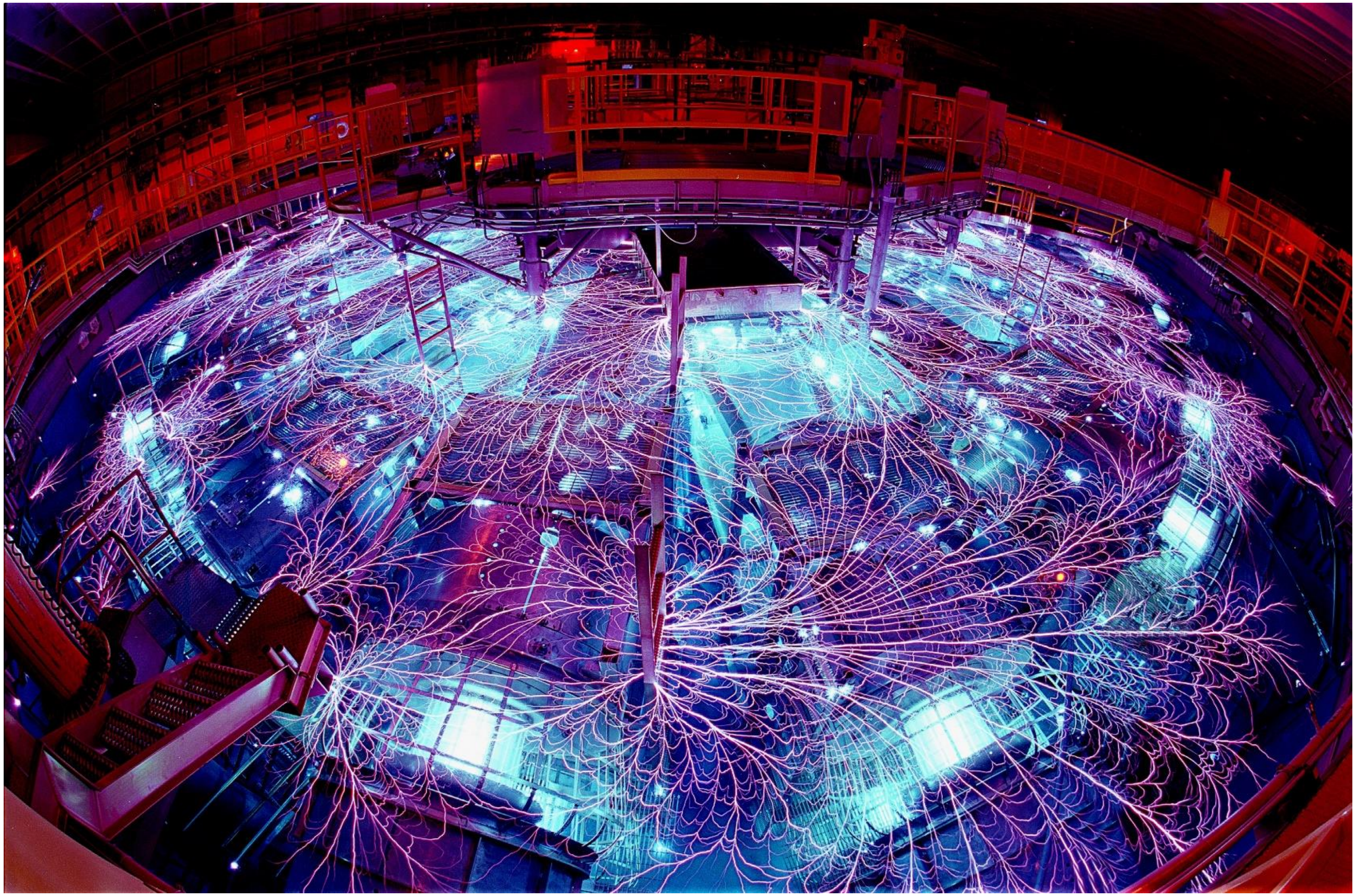
- **Capacitive storage is more common and easier to operate.**

A pulsed power machine at Sandia National Laboratories delivers a 20 MA, 3 MV, and 55-TW pulse

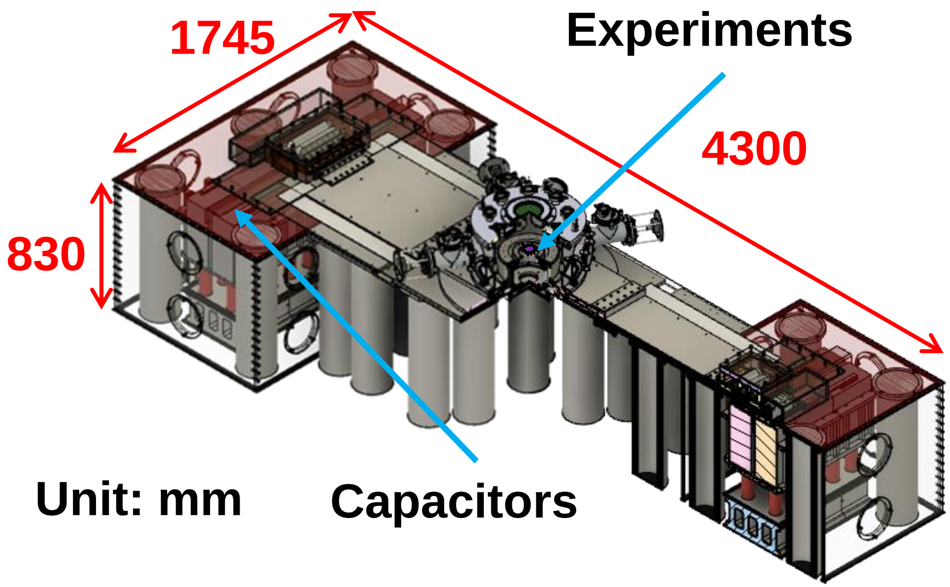


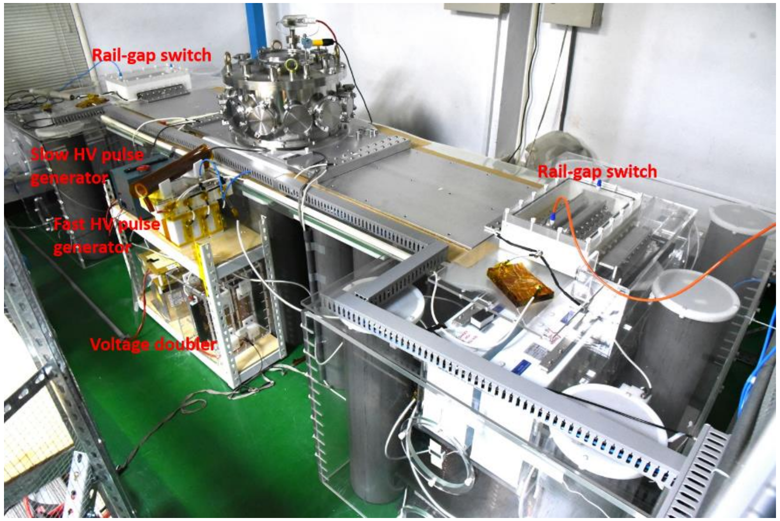
W. A. Stygar *et al.* Phys. Rev. ST Accel. Beams 12, 120401 (2009)

Arcing may happen during the discharge

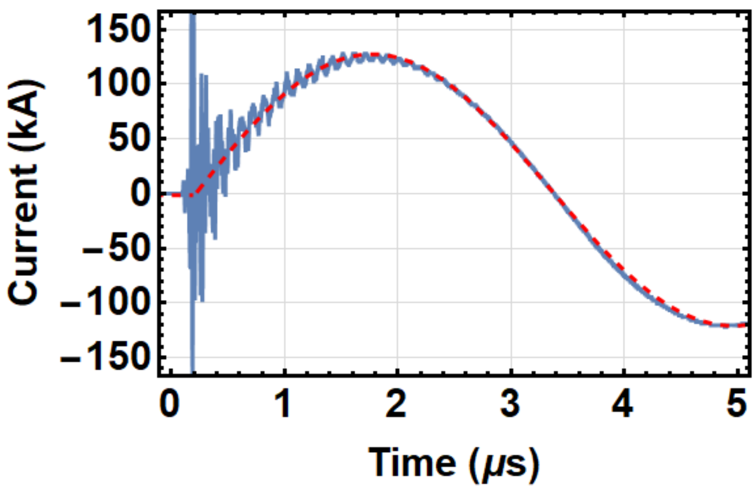


The EUV light source will be developed using the pulsed-power system we built from scratch

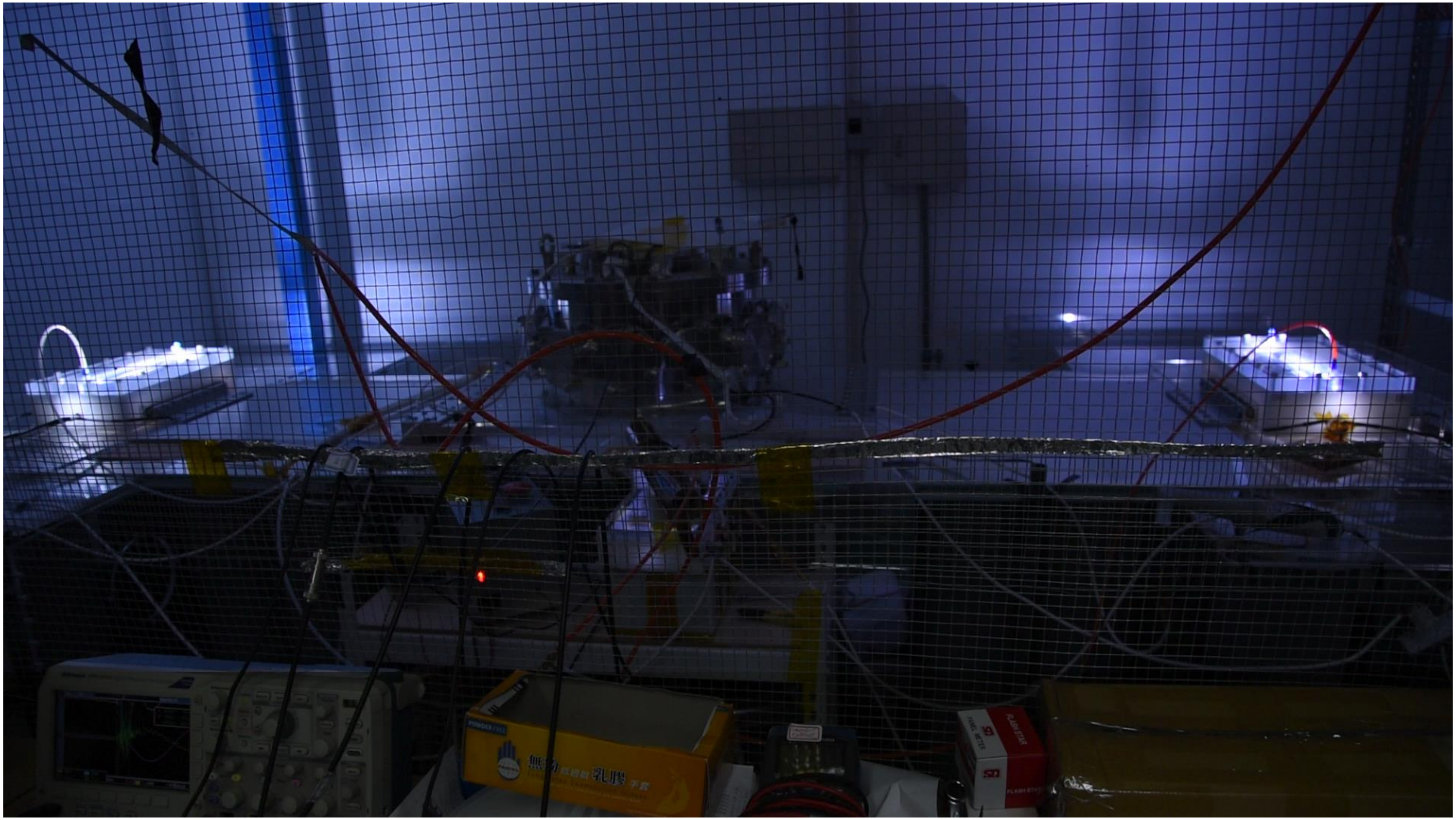




Capacitance (μF)	5	
V_{charge} (kV)	20	(50)
Energy (kJ)	1	(6.25)
Inductance (nH)	204 ± 4	
Rise time (quarter period, ns)	1592 ± 3	
I_{peak} (kA)	135 ± 1	(~340)

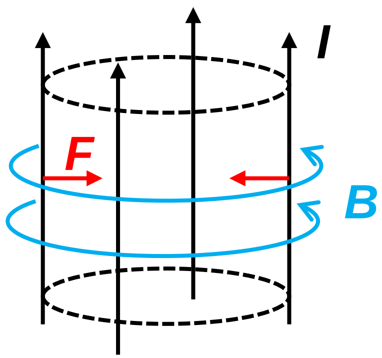


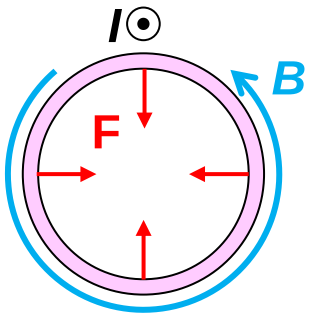
Discharge test in Pulsed-Power Generator for Space Science (PGS) laboratory



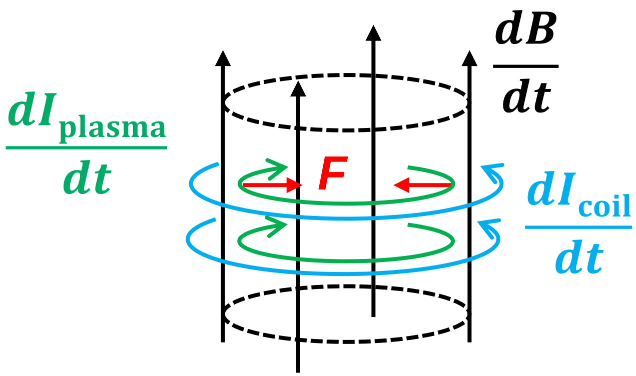
Plasma can be compressed by using jxB force

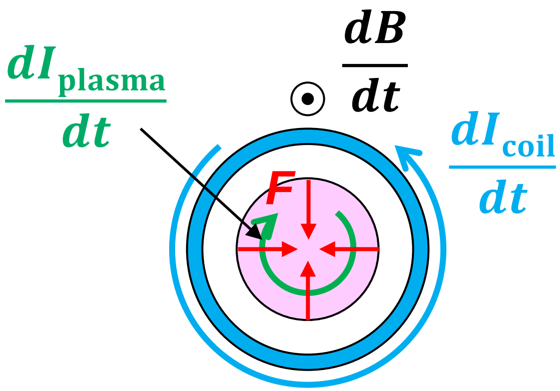
- Z pinches





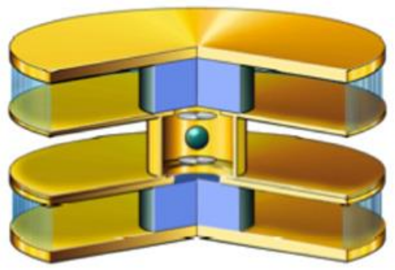
- Theta pinches



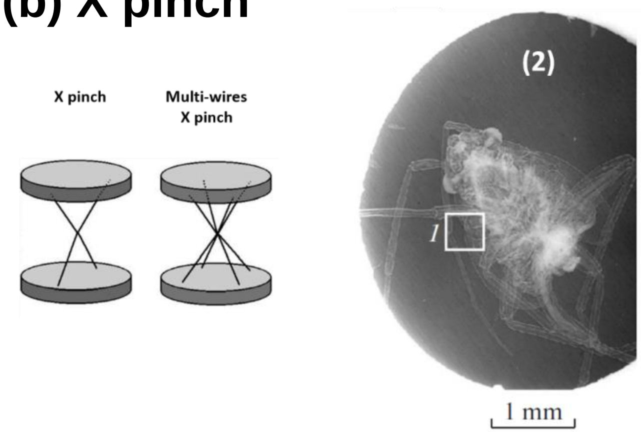


There are multiple applications using pulsed power machine

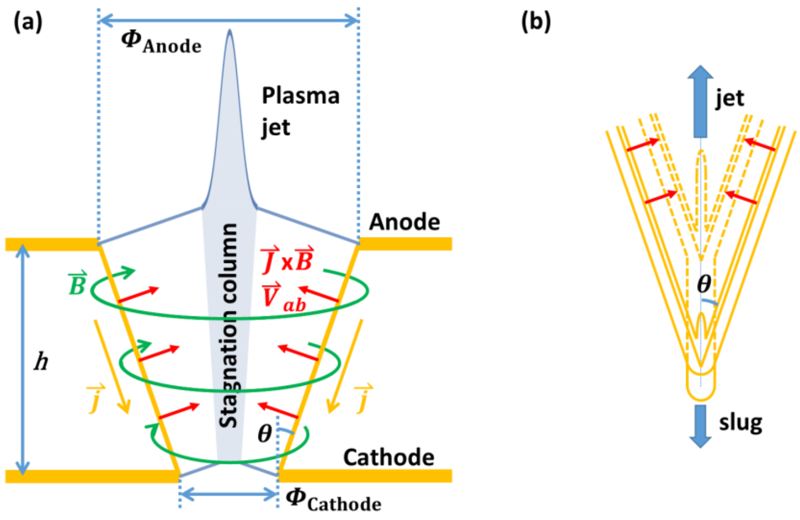
(a) Inertial confinement fusion



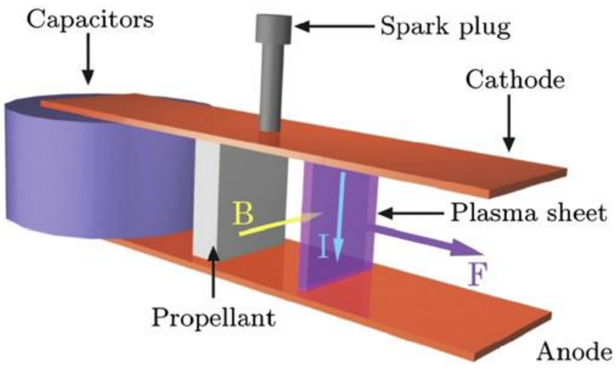
(b) X pinch



(c) Laboratory astrophysics



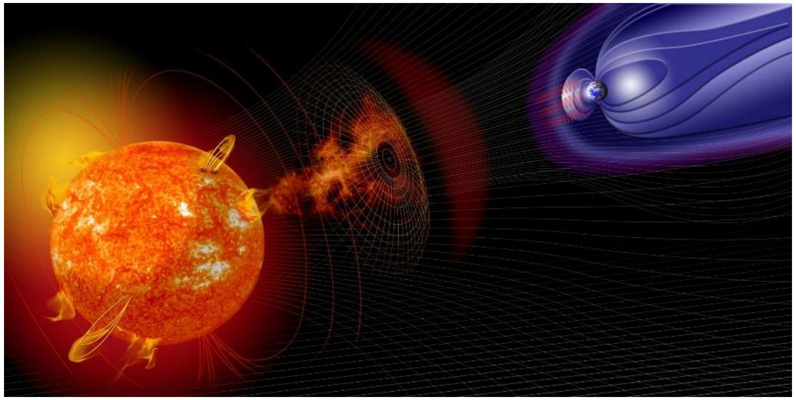
(d) Pulsed plasma thruster



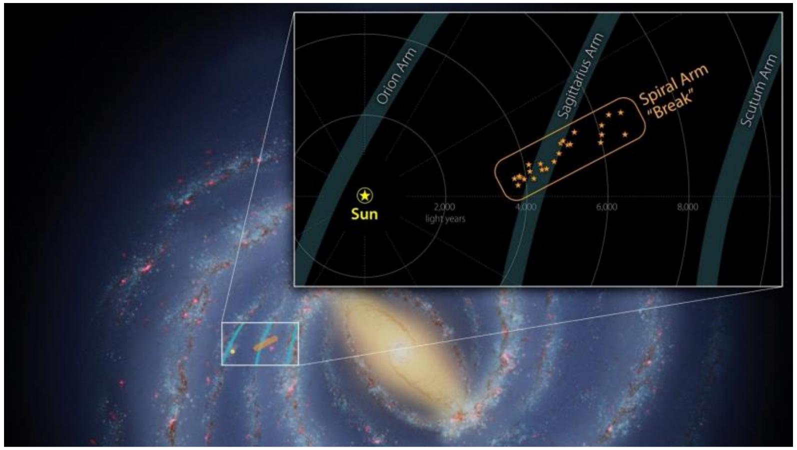
M. G. Haines, Plasma Phys. Control. Fusion 53, 093001 (2011)
G. Birkhoff, etc., J. of Appl. Phys. 19, 563 (1948)
J. Bio. Sci. and Eng. 8, 747 (2015)
Plasma Phys. Report, 42, 226, (2016)
Acta Astronautica 67, 440 (2010)

The pulsed-power system can be used to study laboratory astrophysics and space sciences

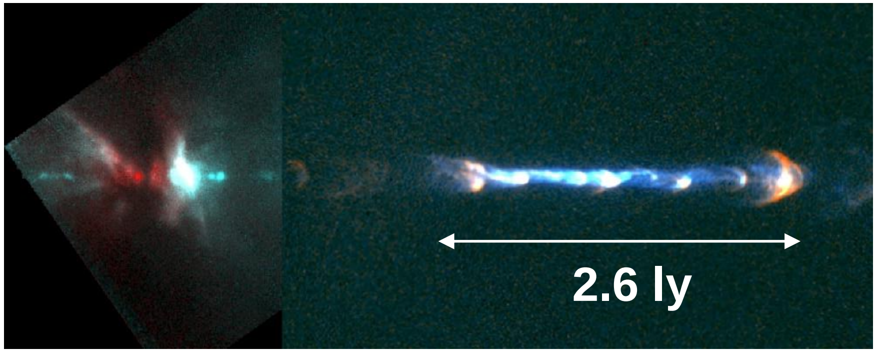
- **Solar wind**



- **Milky way's spiral arms**



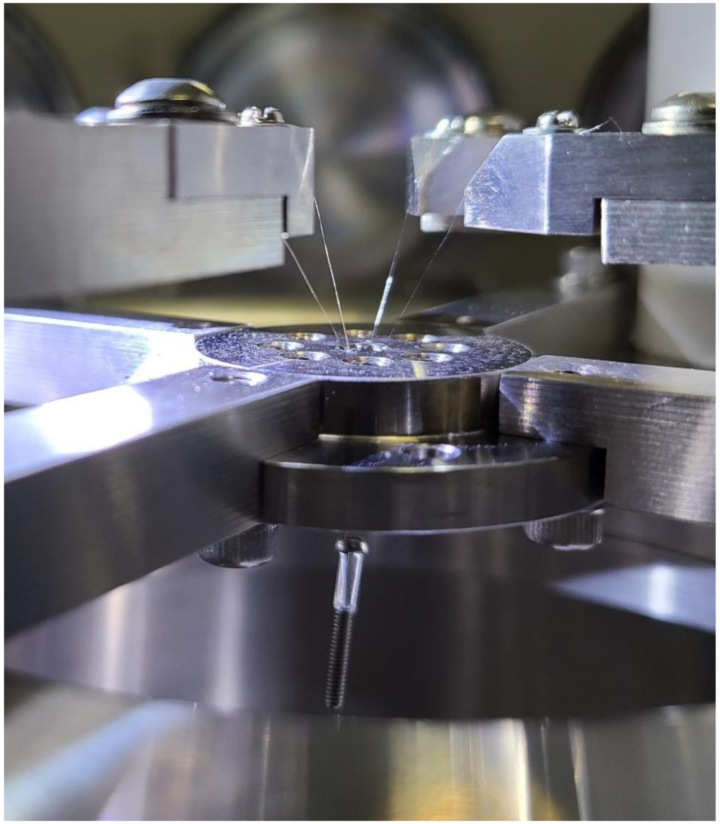
- **Plasma jets e.g., Herbig-Haro object 111**

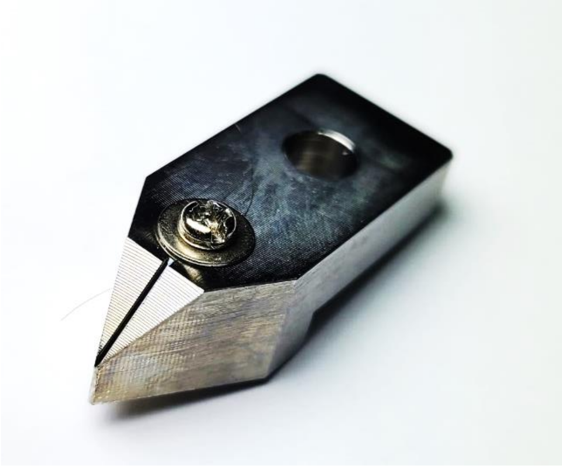


- https://www.nasa.gov/mission_pages/sunearth/spaceweather/index.html
- Jet Propulsion Laboratory [NASA/JPL] Astronomers Find a 'Break' in One of the Milky Way's Spiral Arms (Aug 17, 2021)
- B. Reipurth and J. BallyAnnu, Herbig-Haro Flows: Probes of Early Stellar Evolution. Rev. Astron. Astrophys., 39:403-455, September 2001

The conical-wire array we used in our experiments

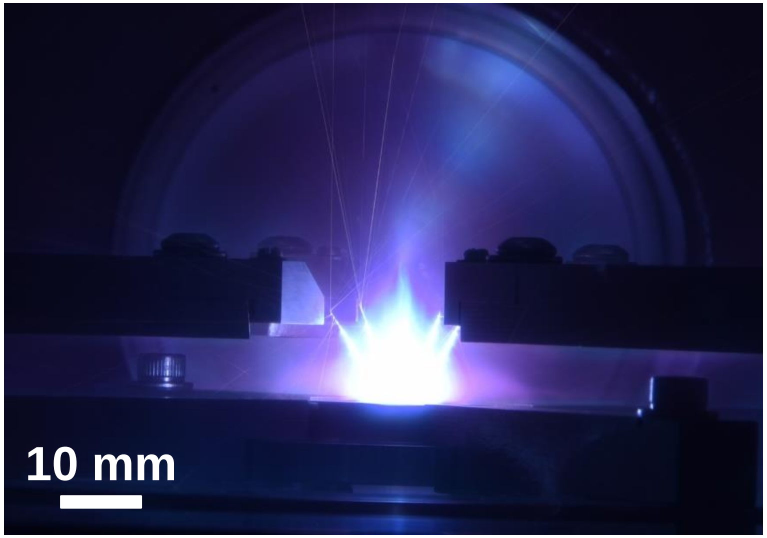
- Material : Tungsten
- Number of wires : 4
- Diameter : 0.02 mm

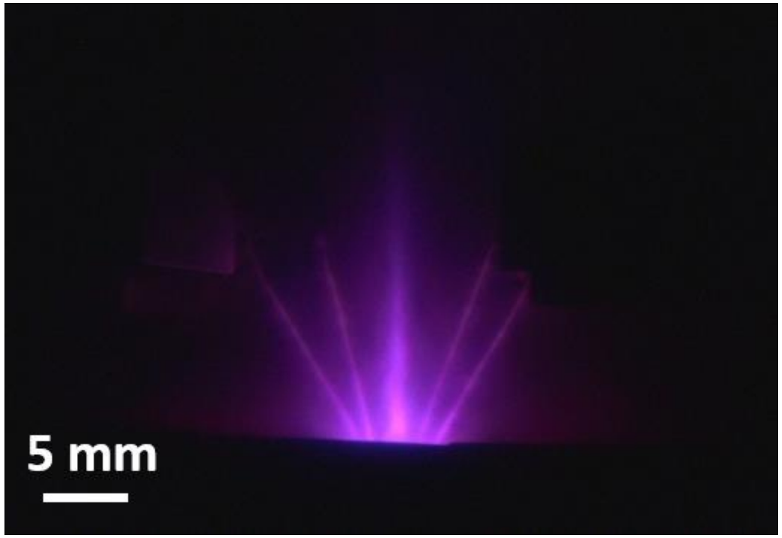




Plasma jets were captured by time-integrated camera in the visible light

- One layer of cellophane with 8 % transmission.
- Three layers of cellophane with 1 % transmission.

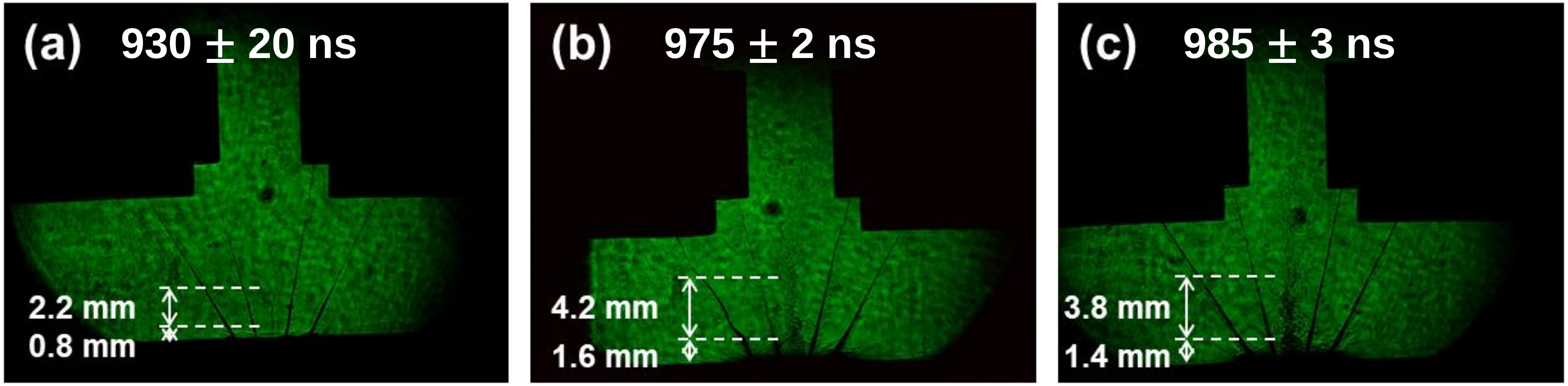




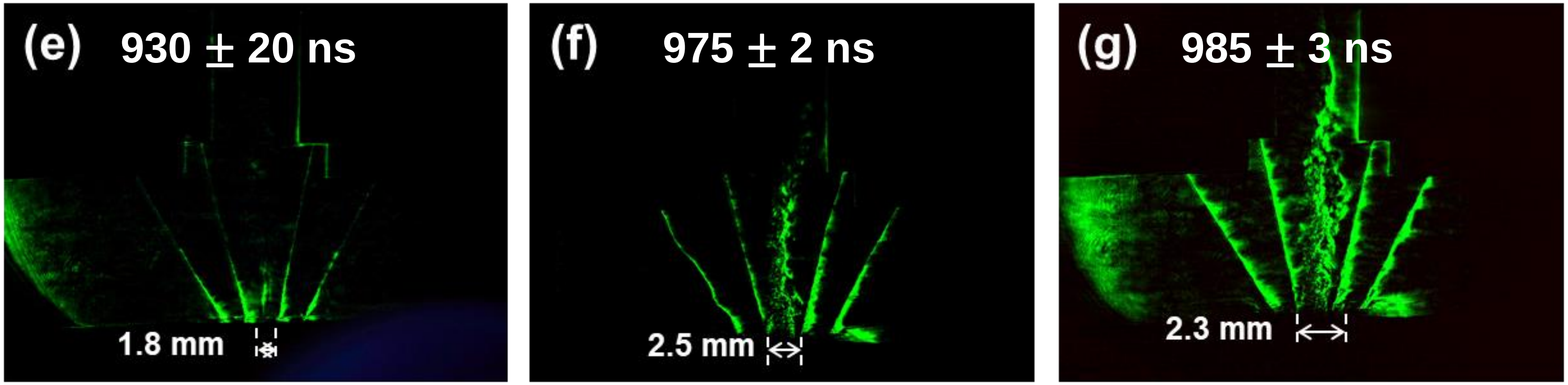
- Both images were taken by using Nikon D750 camera with D/# equals to 22 and ISO= 50 (effective). Exposure times were both 30 sec for being synchronized with the driver.

Time-resolved images show how the plasma plume was generated

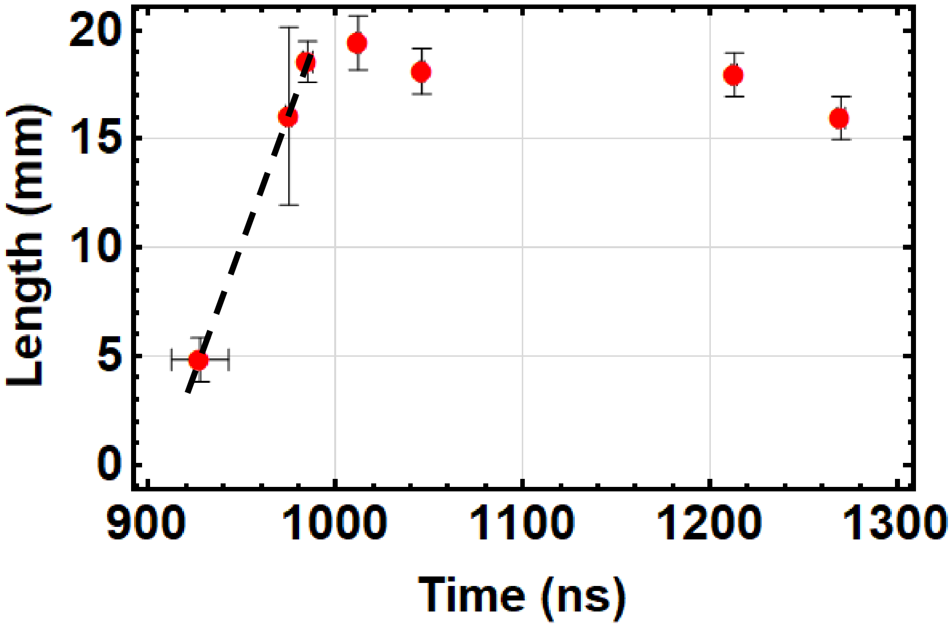
- **Shadowgraph images:**



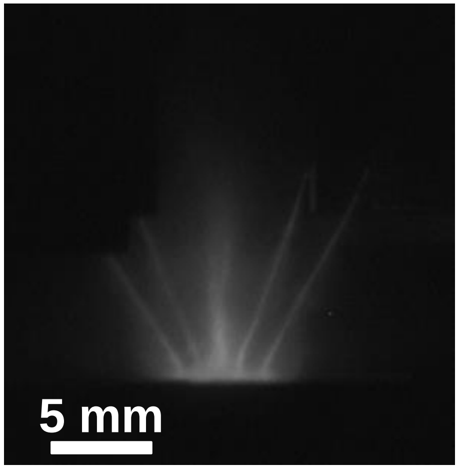
- **Schlieren images:**

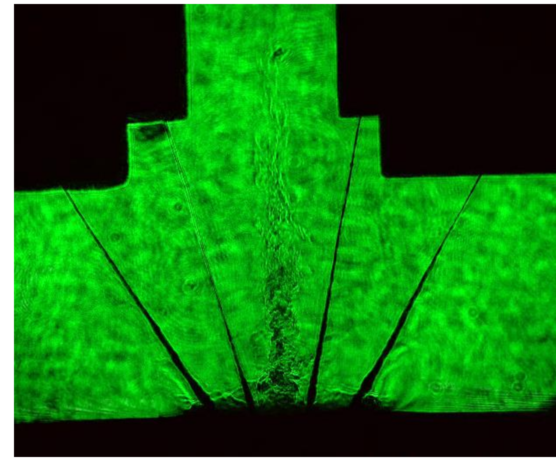


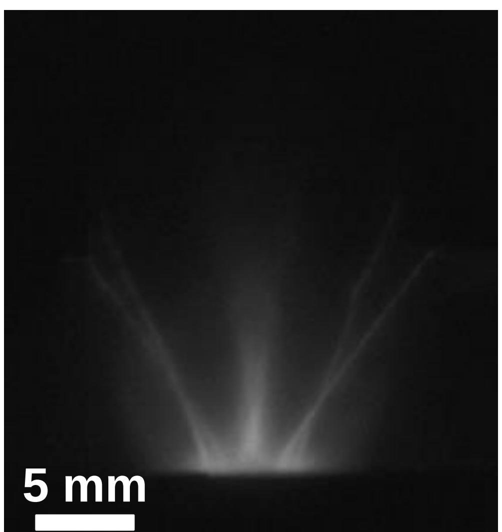
A speed of 170 ± 70 km/sec was estimated

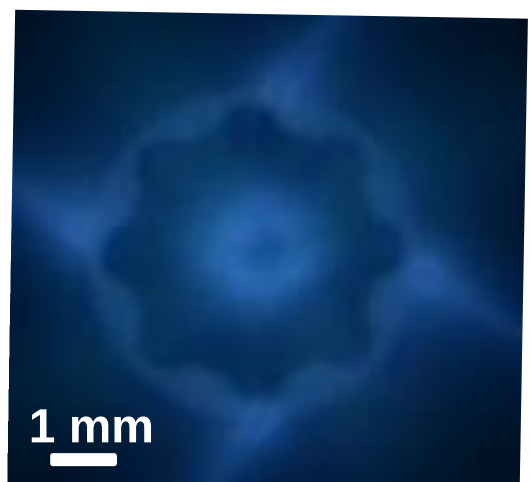


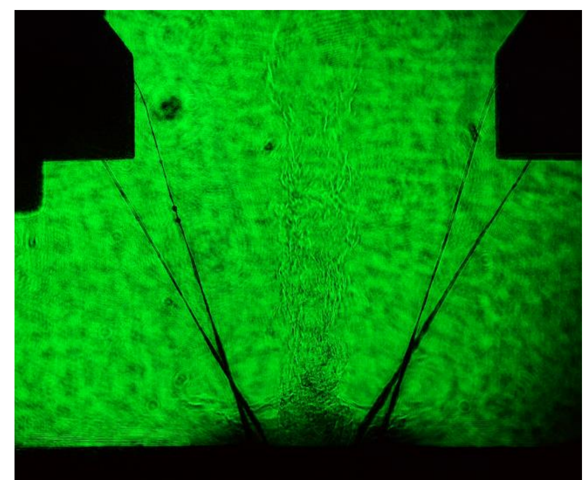
A “tornado” is generated by the twisted conical-wire array





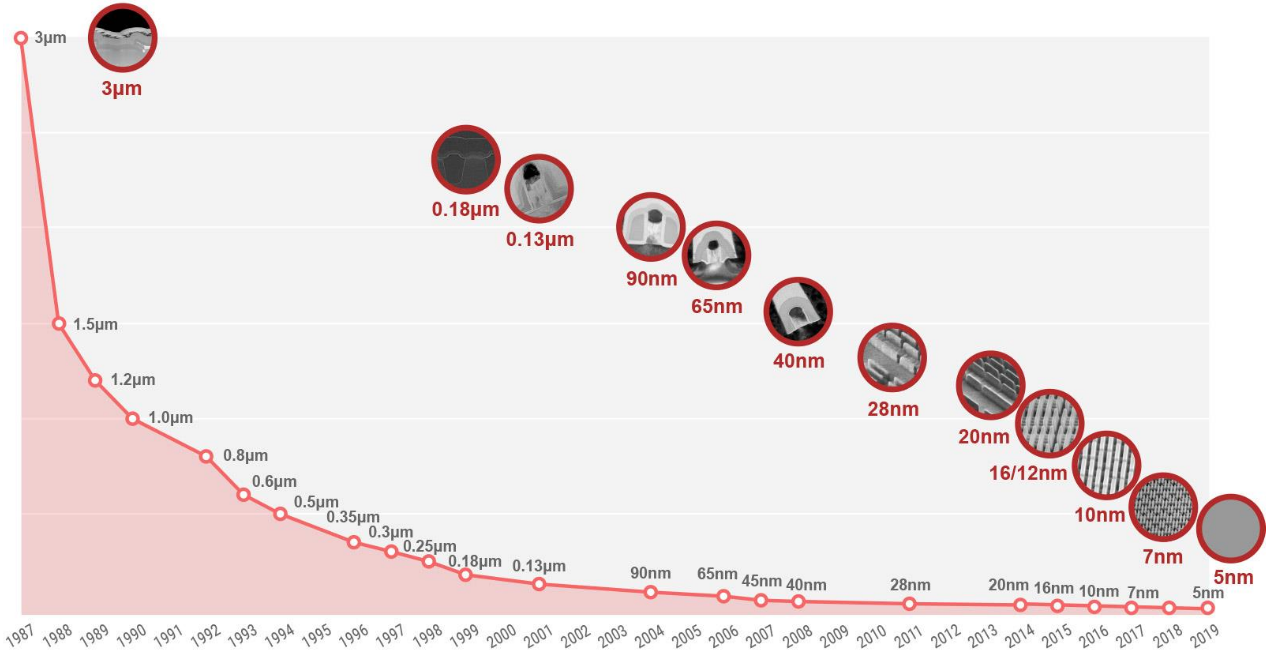






Ultraviolet lithography (EUVL) is one of the key technologies in semiconductor manufacturing nowadays

- **The process technology of Taiwan Semiconductor Manufacturing Company Limited (TSMC):**

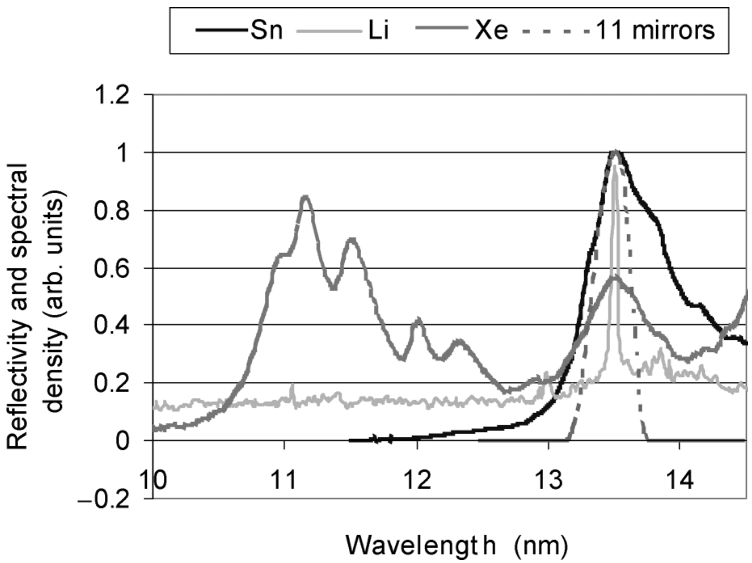


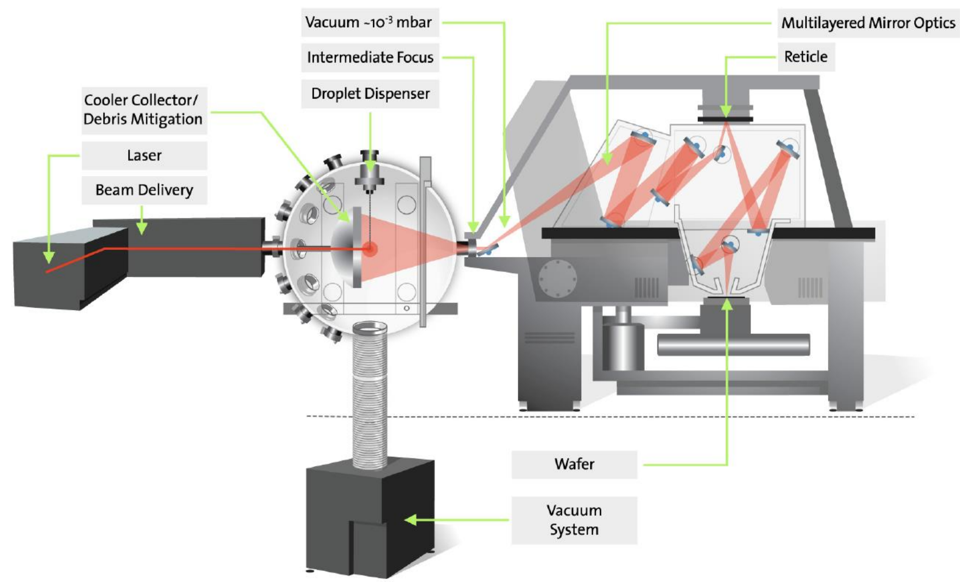
- **Optical diffraction needs to be taken into account.**
- **Shorter wavelength is preferred.**

- **Light source with a center wavelength of 13.5 nm is used.**

https://www.tsmc.com/chinese/dedicatedFoundry/technology/logic.htm

EUV light is generated from laser-produced plasma (LPP)



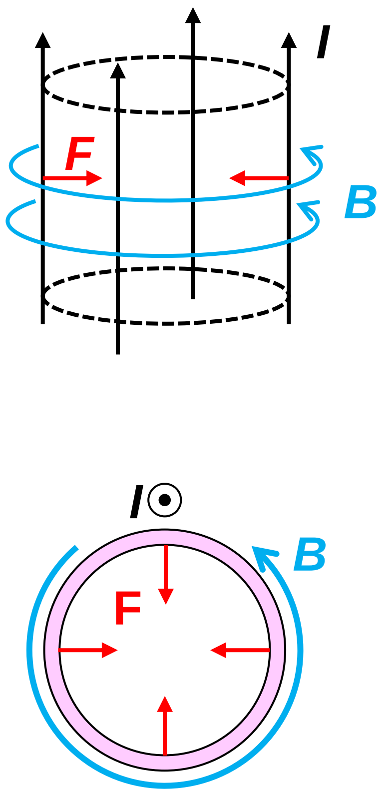


- $\lambda = 13.5\ \mathrm{nm} \pm 1\%$ **is required.**
- **At T=35-40 eV (~450,000 K), in-band emission occurs.**
- **Xenon:**
  - ***$4p^6 4d^8 \rightarrow 4p^6 4d^7 5p$*** **from single ion stage Xe^{10+}**
  - **UTA @ 11 nm**
- **Tin:**
  - ***$4p^6 4d^N \rightarrow 4p^5 4d^{N+1} + 4p^6 4d^{N-1} 4f$*** **($1 \leq N \leq 6$) in ions ranging from Sn^{8+} to Sn^{12+}**
  - **UTA @ 13.5 nm**
- UTA: unresolved transition array

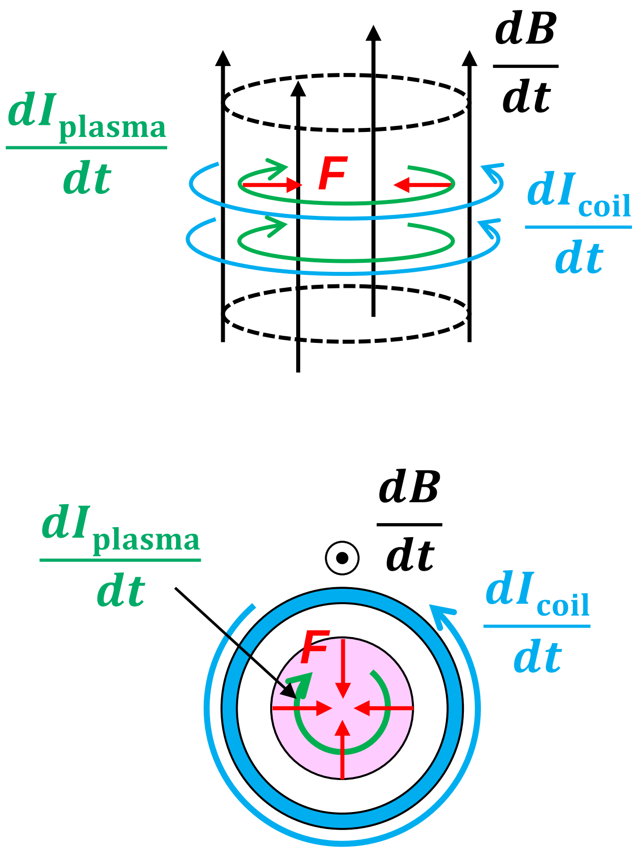
V. Bakshi, EUV sources for lithography
R. S. Abhari, etc., J. Micro/Nanolithography, MEMS, and MOEMS, 11, 021114 (2012)

Plasma can be compressed by using jxB force

- Z pinches

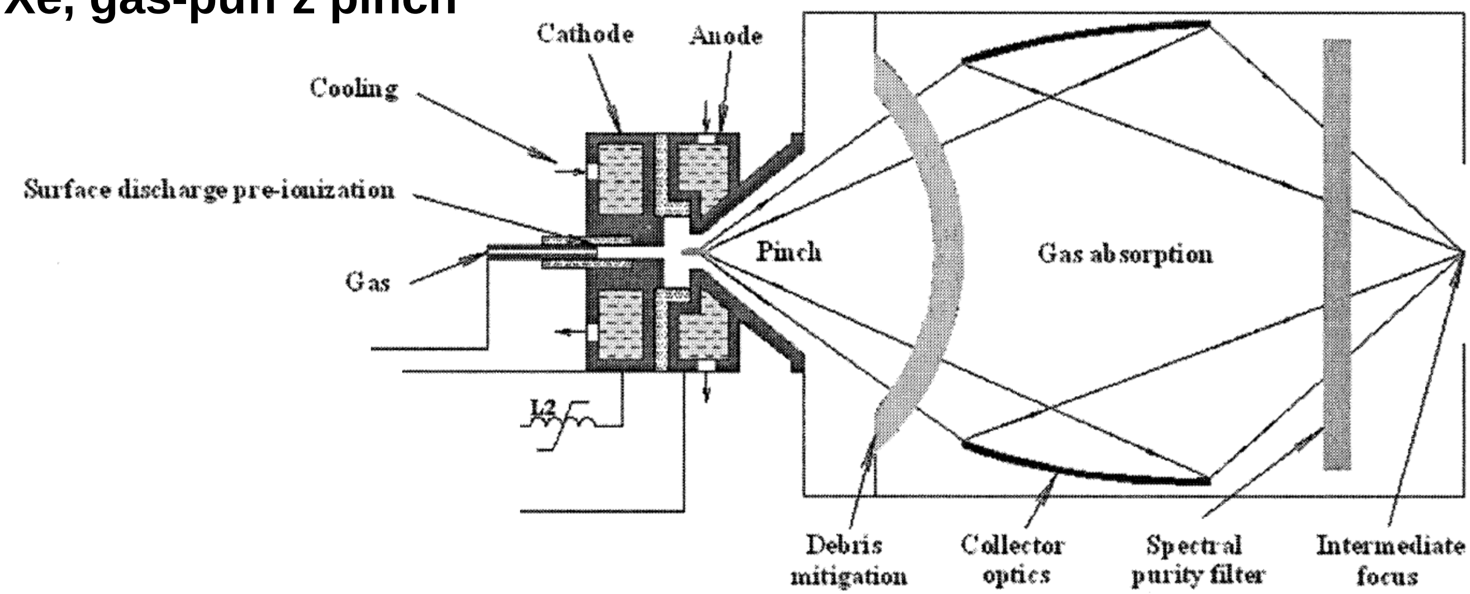


- Theta pinches



Discharge produced plasma (DPP) can generate EUV light for EUV lithography

- **DPP – Xe, gas-puff z pinch**



- **Electrodes are damaged significantly due to the heat and sputtering by ions.**

	Laser-produced plasma (LPP)	Discharge-produced plasma (DPP)
Pros	Commercial system available.	High conversion efficiency.
Cons	Low conversion efficiency.	Short system life time due to electrode erosion.

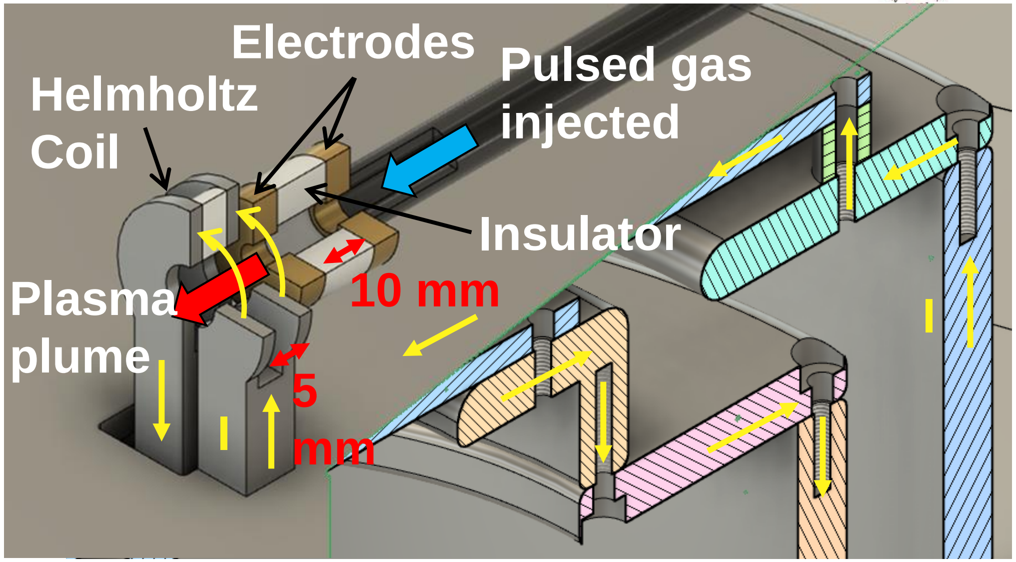
V. Borisov, etc., Proc. SPIE 6611, Laser Optics 2006: High-Power Gas Lasers, 66110B (12 April 2007)

EUV light characteristics will be measured.

- **Two steps of the development:**

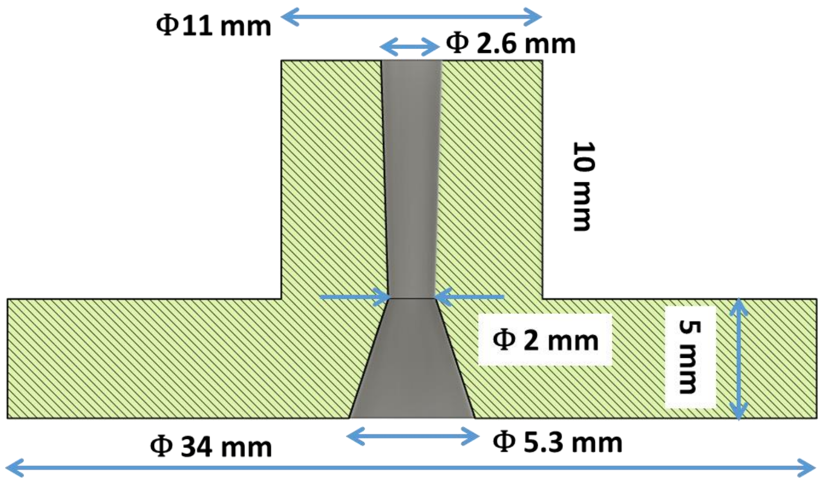
  (1) Studies the plasma plume.

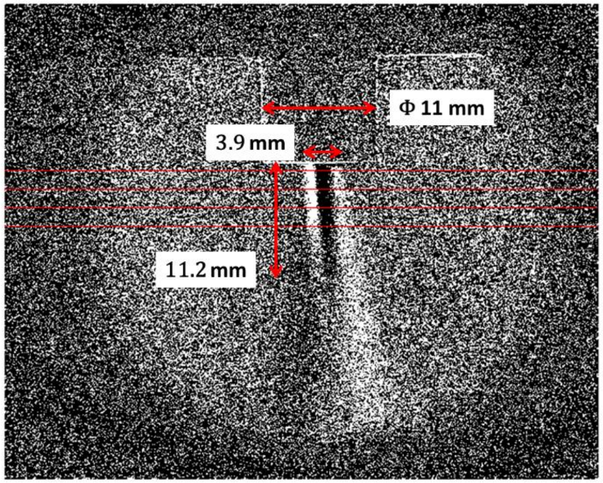
  (2) Studies the theta pinch.



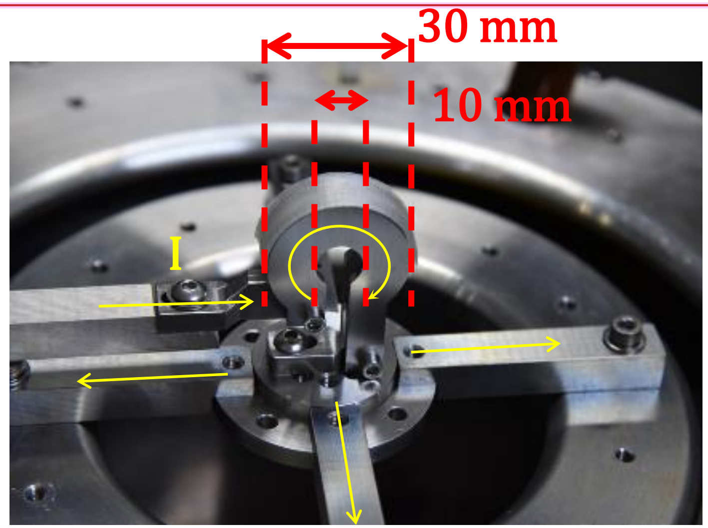
- **Characteristics of the initial plasma plume will be measured.**
- **Plasma density, temperature before and after compression will be measured.**
- **EUV light characteristic will be measured.**
  - **Intensity**
  - **Pulse width**
  - **Spectrum**
  - **Uniformity**
  - **…….**

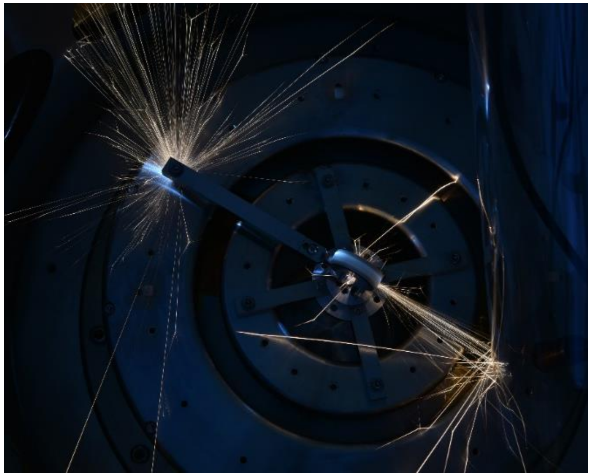
Supersonic gas puff will be generated by using convergent-divergent nozzle

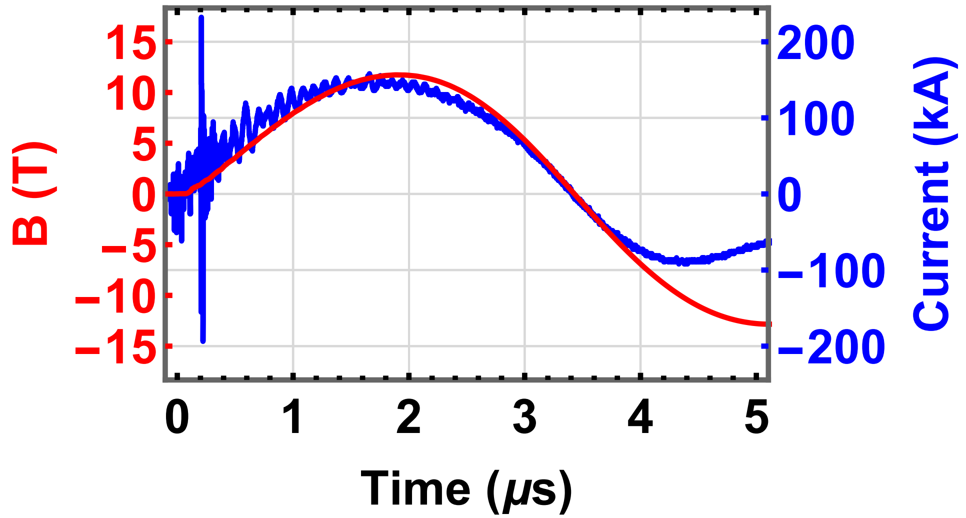




Driving the coil is pretty exciting







- **The platform can be used to study Magnetized Target Fusion (MTF).**

Outlines

- Introduction to pulsed-power system
- **Review of circuit analysis**
- Static and dynamic breakdown strength of dielectric materials
  - Gas – Townsend discharge (avalanche breakdown), Paschen's curve
  - Liquid
  - Solid
- Energy storage
  - Pulse discharge capacitors
  - Marx generators
  - Inductive energy storage

Kirchhoff's current law

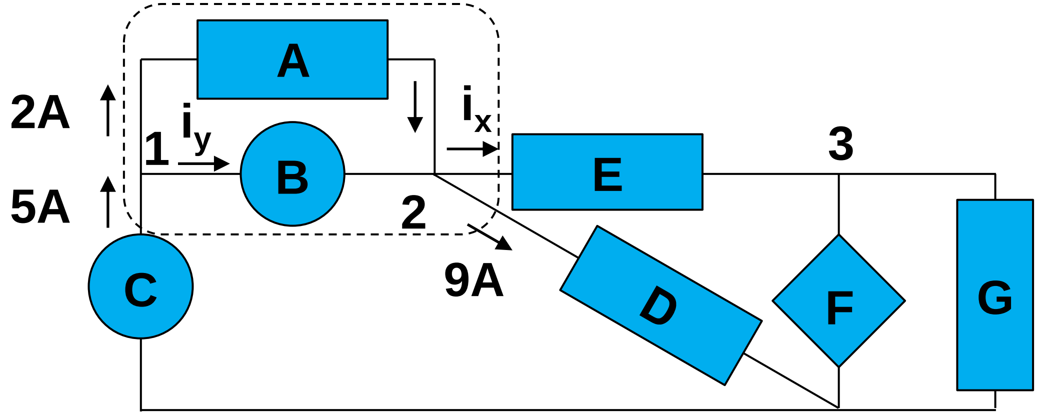
- **At any instant in time, the algebraic sum of all currents leaving any closed surface is zero.**

$$\boldsymbol{i_1 + i_2 + \ldots + i_N = 0}$$

or in abbreviated notation:

$$\sum_{k=1}^{N} \boldsymbol{i_k = 0}$$

where i_k is the k^{th} current of the N currents leaving the closed surfaces.



$$\boldsymbol{i_y + 2 - 5 = 0}$$

$$\boldsymbol{i_y = 3(A)}$$

$$\boldsymbol{-5 + i_x + 9 = 0}$$

$$\boldsymbol{i_x = -4(A)}$$

$$\boldsymbol{2 + i_y - i_x - 9 = 0}$$

Kirchhoff’s voltage law

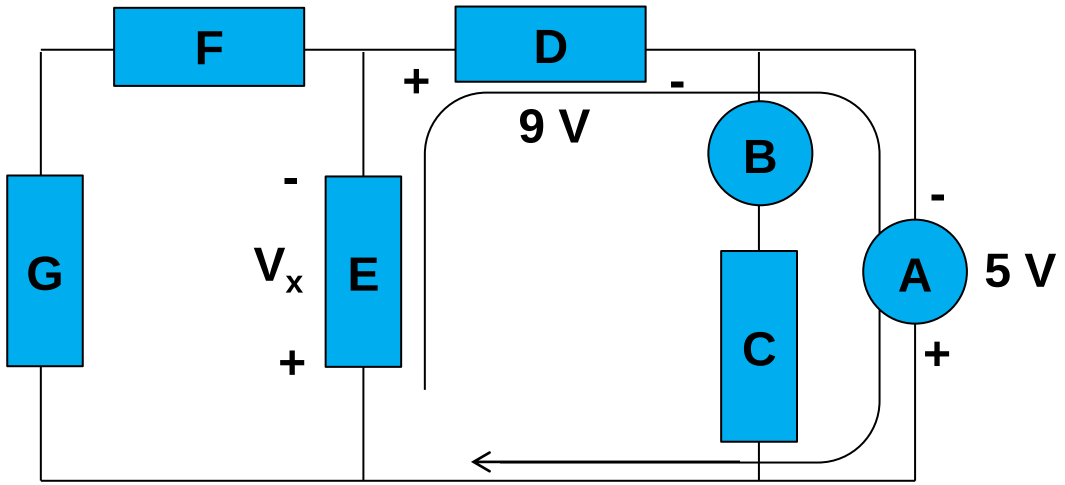
- **At any instant in time, the algebraic sum of all voltage drops taken around any closed path is zero.**

$$V_1 + V_2 + ... + V_N = 0$$

or in abbreviated notation:

$$\sum_{k=1}^{N} V_k = 0$$

where V_k is the voltage drop, taken in the direction of the path along the k^{th} segment of the N segments in the closed path.

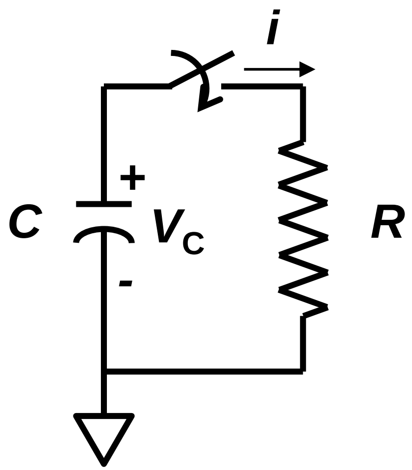


$$-V_x - 9 + 5 = 0$$

$$V_x = -4 \text{ (V)}$$

Source-free RC circuit

- Assuming that the capacitor is fully charged to V_0.
- At $t=0^+$, the switch is closed.



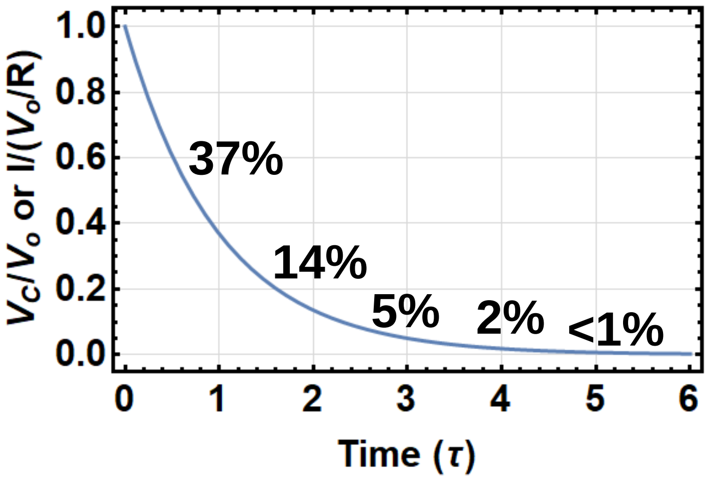
$$V_C - iR = 0 \qquad i = \frac{dQ}{dt} = -C\frac{dV_C}{dt}$$

$$V_C + RC\frac{dV_C}{dt} = 0 \qquad \frac{dV_C}{dt} + \frac{1}{RC}V_C = 0$$

$$\int_{V_o}^{V_C(t)} \frac{1}{V_C}\,dV_C = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{V_C(t)}{V_o}\right) = -\frac{t}{RC}$$

$$V_C(t) = V_o e^{-t/RC} \equiv V_o e^{-t/\tau_C} \qquad \tau_C \equiv RC$$



$$I(t) = -C\frac{dV_C}{dt} = -V_oC\left(-\frac{1}{\tau_C}\right)e^{-t/\tau_C} = \frac{V_o}{R}e^{-t/\tau_C}$$

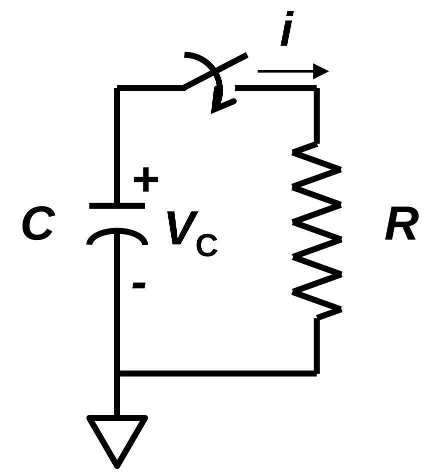
Bleeder resistors dissipate energy in the capacitor for safety

- **Example 1:**

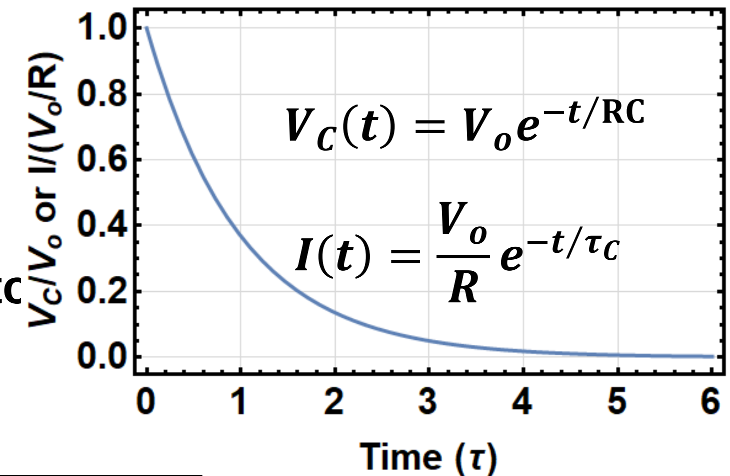
  V_o=50 kV , C=1 μF, V≤ 10 V is safe.

  If the bleeder resistor takes 15 mins to dissipate energy in the capacitor, then

$$10 = 50\text{k Exp}\left(-\frac{15\times 60}{R\times 10^{-6}}\right) \qquad R = 10\text{M}\Omega$$

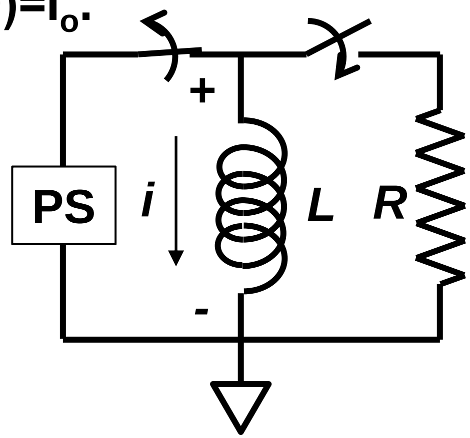


- **Example 2: SOP for working on high voltage system.**
  - **1st chicken stick with a large resistors is needed to dissipate the energy in the capacitor slowly first.**
  - **2nd chicken stick that ground the capacitc is needed after most of the energy is dumped.**



Source-free RL circuit

- Assuming that the current is at steady state for t ≤0, I(0)=I_o.
- At t=0^+, the switch is opened/closed.



$$-\mathrm{IR} - V_L = 0 \qquad V_L = L\frac{\mathrm{dI}}{\mathrm{dt}}$$

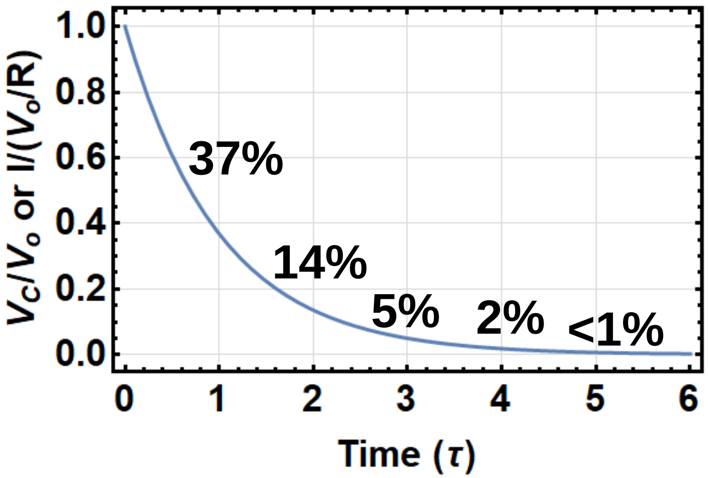
$$\mathrm{IR} + L\frac{\mathrm{dI}}{\mathrm{dt}} = 0 \qquad \frac{dI}{\mathrm{dt}} + \frac{R}{L}I = 0$$

$$\int_{I_o}^{I(t)} \frac{1}{I}\mathrm{d}I = -\frac{\mathrm{R}}{\mathrm{L}}\int_0^t \mathrm{d}t$$

$$\ln\left(\frac{I(t)}{I_o}\right) = -\frac{R}{L}t \equiv \frac{t}{\tau_L} \qquad \tau_L \equiv \frac{L}{R}$$

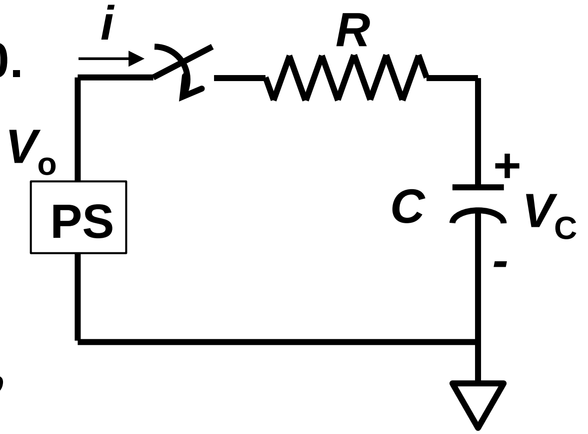
$$I(t) = I_o e^{-\frac{R}{L}t} \equiv I_o e^{-t/\tau_L}$$

$$V(t) = L\frac{dI}{dt} = \mathrm{LI}_o\left(-\frac{1}{\tau_L}\right)e^{-t/\tau_L} = -\mathrm{RI}_o e^{-t/\tau_L}$$



Charging of a capacitor

- **Assuming that the capacitor is NOT charged at t≤0.**
- **At $t=0^+$, the switch is closed.**



$$V_o - iR - V_C = 0 \qquad i = \frac{dQ}{dt} = C\frac{dV_C}{dt}$$

$$V_o - RC\frac{dV_C}{dt} - V_C = 0 \qquad \frac{dV_C}{dt} = -\frac{1}{RC}V_C + \frac{1}{RC}V_o$$

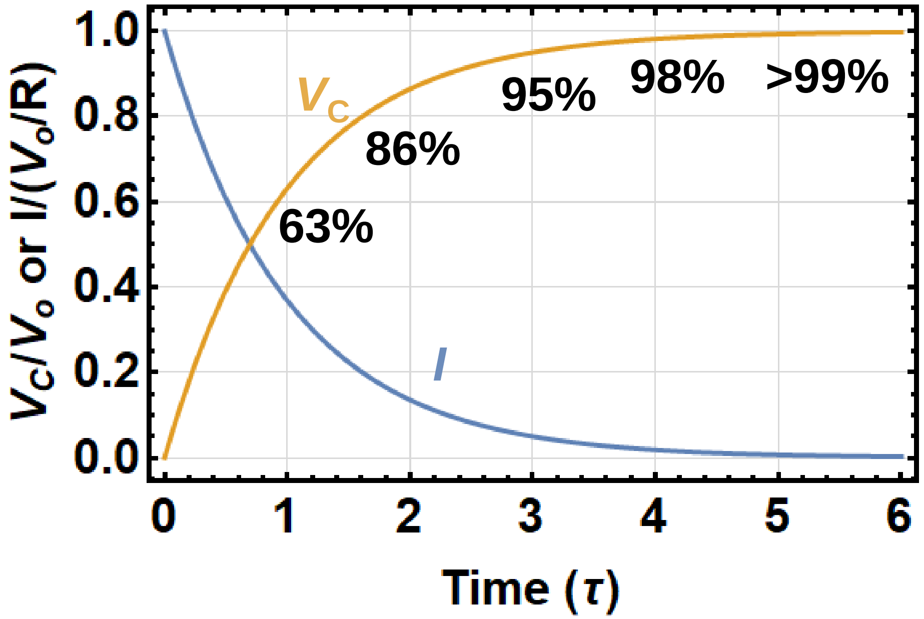
$$\frac{d(V_C - V_o)}{dt} = -\frac{1}{RC}(V_C - V_o) \qquad V' \equiv V_C - V_o \qquad V'(t=0) = -V_o$$

$$\frac{dV'}{dt} = -\frac{1}{RC}V' \qquad \int_{-V_o}^{V'(t)} \frac{1}{V'}\, dV' = -\frac{1}{RC}\int_0^t dt \qquad \ln\left(\frac{V'(t)}{-V_o}\right) = -\frac{t}{RC}$$

$$V'(t) = -V_o e^{-t/\tau_C} \qquad V_C = V_o\left(1 - e^{-t/\tau_C}\right) \qquad \tau_C \equiv RC$$

$$I(t) = C\frac{dV_C}{dt} = -V_o\left(-\frac{1}{\tau_C}\right)e^{-t/\tau_C} = \frac{V_o}{R}e^{-t/\tau_C}$$

The capacitor is almost fully charged after 5 time constant

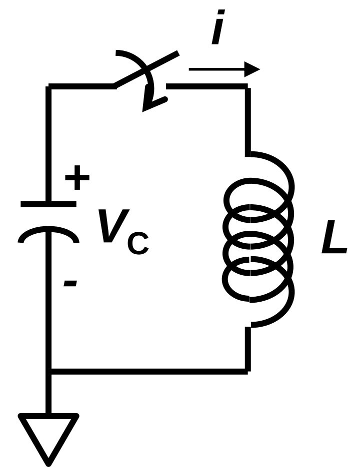


$$V_C = V_o\left(1 - e^{-t/\tau_C}\right)$$

$$I(t) = \frac{V_o}{R} e^{-t/\tau_C}$$

$$\tau_C \equiv \mathrm{RC}$$

LC oscillation



- **Assuming that the capacitor is fully charged to V_0, I(0)=0.**
- **At t=0⁺, the switch is closed.**

$$V_C - V_L = 0 \qquad i = \frac{dQ}{dt} = -C\frac{dV_C}{dt} \qquad V_L = L\frac{di}{dt} = -LC\frac{d^2V_C}{dt^2}$$

$$\frac{d^2V_C}{dt^2} + \frac{1}{LC}V_c = 0$$

$$V_C(t) = \alpha\sin(\omega t) + \beta\cos(\omega t) \qquad \omega \equiv \frac{1}{\sqrt{LC}}$$

$$i = -C\big(\alpha\omega\cos(\omega t) - \beta\omega\sin(\omega t)\big)$$

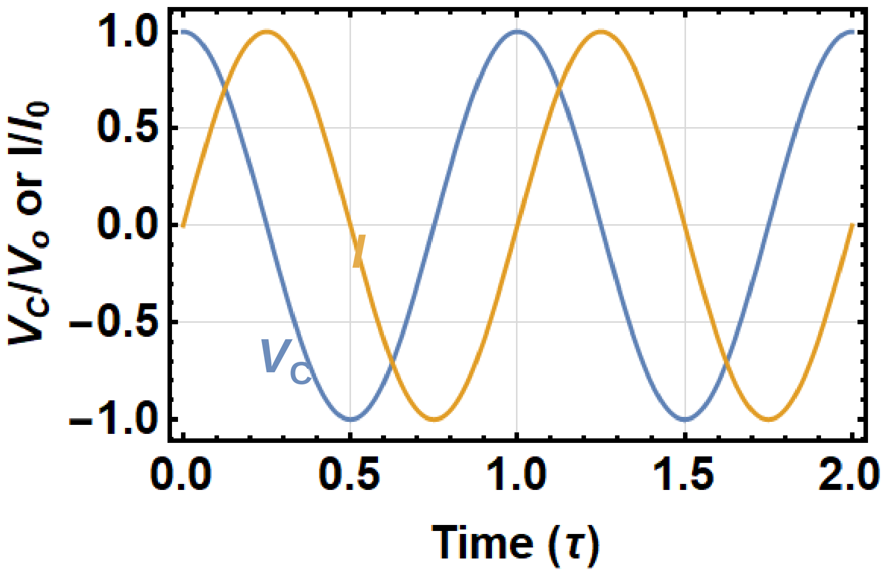
$$i(t=0) = 0 = -C\alpha\omega \qquad \alpha = 0$$

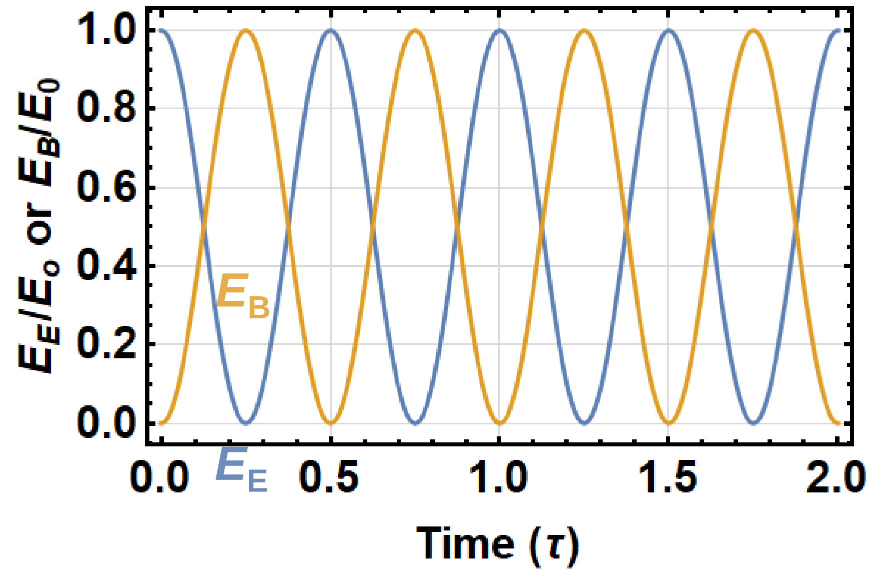
$$V_C(t=0) = V_o = \beta$$

$$V_C = V_o\cos(\omega t)$$

$$i = \frac{V_o}{\sqrt{L/C}}\sin(\omega t)$$

Energy is oscillating between the capacitor and the inductor





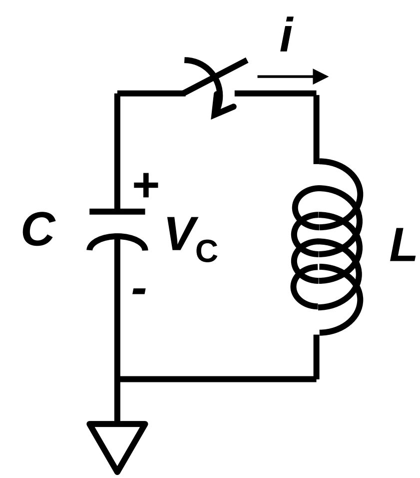
$$V_C = V_o \cos(\omega t)$$

$$i = \frac{V_o}{\sqrt{L/C}} \sin(\omega t)$$

$$\omega \equiv \frac{1}{\sqrt{LC}}$$

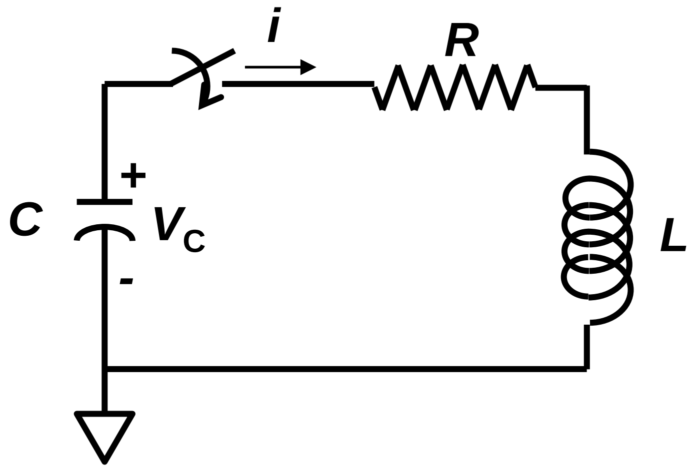
$$E_E = \frac{1}{2} C V_C{}^2$$

$$E_B = \frac{1}{2} L i^2$$



Series RLC circuit

- Assuming that the capacitor is fully charged to V_0, I(0)=0.
- At $t=0^+$, the switch is closed.



$$V_C - \mathrm{iR} - V_L = 0 \qquad i = \frac{\mathrm{dQ}}{\mathrm{dt}} = -C\frac{\mathrm{dV}_C}{\mathrm{dt}}$$

$$V_L = L\frac{\mathrm{di}}{\mathrm{dt}} = -\mathrm{LC}\frac{d^2V_C}{\mathrm{dt}^2}$$

$$\frac{d^2V_C}{\mathrm{dt}^2} + \frac{R}{L}\frac{\mathrm{dV}_C}{\mathrm{dt}} + \frac{1}{\mathrm{LC}}V_c = 0$$

$$D^2 + \frac{R}{L}D + \frac{1}{\mathrm{LC}} = 0 \qquad D = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{\mathrm{LC}}}$$

$$V_C = \exp\left(-\frac{R}{2L}t\right)\left[\alpha\exp\left(\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{\mathrm{LC}}}\,t\right) + \beta\exp\left(-\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{\mathrm{LC}}}\,t\right)\right]$$

Underdamped condition

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} < 0$$

$$V_C = \exp\left(-\frac{R}{2L}t\right)\left[\alpha\exp\left(i\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\,t\right)+\beta\exp\left(-i\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\,t\right)\right]$$

$$V_C = \exp\left(-\frac{R}{2L}t\right)\left[\alpha\sin\left(\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\,t\right)+\beta\cos\left(\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\,t\right)\right]$$

$$V_C = \exp\left(-\frac{R}{2L}t\right)[\alpha\cos(\omega t)+\beta\sin(\omega t)] \qquad \omega \equiv \sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$$

$$V_C(0) = \alpha = V_o \qquad V_C(t) = \exp\left(-\frac{R}{2L}t\right)[V_o\cos(\omega t)+\beta\sin(\omega t)]$$

$$i = -C\frac{dV_C}{dt} = \left[-\frac{R}{2L}\exp\left(-\frac{R}{2L}t\right)\left(V_o\cos(\omega t)+\beta\sin(\omega t)\right)\right.$$

$$\left.+\exp\left(-\frac{R}{2L}t\right)\left(-V_o\omega\sin(\omega t)+\beta V_o\omega\cos(\omega t)\right)\right]$$

Underdamped condition

$$I(0) = -C\left(-\frac{R}{2L}V_o + \beta\omega\right) = 0 \qquad \beta = \frac{R}{2L}\frac{V_o}{\omega} = V_0 \frac{R/2\,L}{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}} = \frac{V_o}{\sqrt{\frac{4L}{R^2C} - 1}}$$

$$I = -C\frac{dV_C}{dt} = \left[-\frac{R}{2L}\exp\left(-\frac{R}{2L}t\right)\left(V_o\cos(\omega t) + \frac{R}{2L}\frac{V_o}{\omega}\sin(\omega t)\right)\right.$$
$$\left. + \exp\left(-\frac{R}{2L}t\right)\left(-V_o\omega\sin(\omega t) + \frac{R}{2L}\frac{V_o}{\omega}\omega\cos(\omega t)\right)\right]$$

$$i(t) = \frac{V_o}{\sqrt{\frac{L}{C} - \left(\frac{R}{2}\right)^2}}\exp\left(-\frac{R}{2L}t\right)\sin(\omega t)$$

$$V_C(t) = V_o\exp\left(-\frac{R}{2L}t\right)\left[\cos(\omega t) + \frac{R/2\,L}{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}\sin(\omega t)\right]$$

Overdamped condition

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{\mathrm{LC}} > 0$$

$$V_C = \exp\left(-\frac{R}{2L}t\right)\left[\alpha\exp\left(\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{\mathrm{LC}}}\,t\right) + \beta\exp\left(-\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{\mathrm{LC}}}\,t\right)\right]$$

$$V_C = \exp\left(-\frac{R}{2L}t\right)[\alpha\exp(\gamma t) + \beta\exp(-\gamma t)] \qquad \gamma \equiv \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{\mathrm{LC}}}$$

$$V_C(t=0) = V_0 = \alpha + \beta$$

$$i = -C\frac{\mathrm{d}V_C}{\mathrm{dt}} = -\mathrm{C}\left\{-\frac{R}{2L}\exp\left(-\frac{R}{2L}t\right)[\alpha\exp(\gamma t) + \beta\exp(-\gamma t)] + \exp\left(-\frac{R}{2L}t\right)[\alpha\gamma\exp(\gamma t) - \beta\gamma\exp(-\gamma t)]\right\}$$

Overdamped condition

$$i(0) = -C\left[-\frac{R}{2L}(\alpha+\beta) + (\alpha\gamma - \beta\gamma)\right] = 0$$

$$\alpha\left(\gamma - \frac{R}{2L}\right) - \beta\left(\gamma + \frac{R}{2L}\right) = 0$$

$$\alpha = \frac{\left(\gamma + \frac{R}{2L}\right)}{\left(\gamma - \frac{R}{2L}\right)}\beta$$

$$V_0 = \alpha + \beta$$

$$\beta\left[1 + \frac{\left(\gamma + \frac{R}{2L}\right)}{\left(\gamma - \frac{R}{2L}\right)}\right] = V_o$$

$$\beta = \frac{V_o}{2}\left(1 - \frac{R/2\,L}{\gamma}\right)$$

$$\alpha = \frac{V_o}{2}\left(1 + \frac{R/2\,L}{\gamma}\right)$$

$$V_C = \frac{V_o}{2}\exp\left(-\frac{R}{2L}t\right)\left[\left(1 + \frac{R/2\,L}{\gamma}\right)\exp(\gamma t) + \left(1 - \frac{R/2\,L}{\gamma}\right)\exp(-\gamma t)\right]$$

$$i = \frac{V_o}{2\gamma}\frac{1}{L}\exp\left(-\frac{R}{2L}t\right)[\exp(\gamma t) - \exp(-\gamma t)]$$

$$\gamma \equiv \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Critically damped condition

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{\mathrm{LC}} = 0 \qquad R_{\mathrm{cri}} = 2\sqrt{\frac{L}{C}}$$

$$V_C = (\alpha + \beta t)\exp\left(-\frac{R}{2L}t\right) \qquad V_C = (V_o + \beta t)\exp\left(-\frac{R}{2L}t\right) \qquad V_C(0) = V_0 = \alpha$$

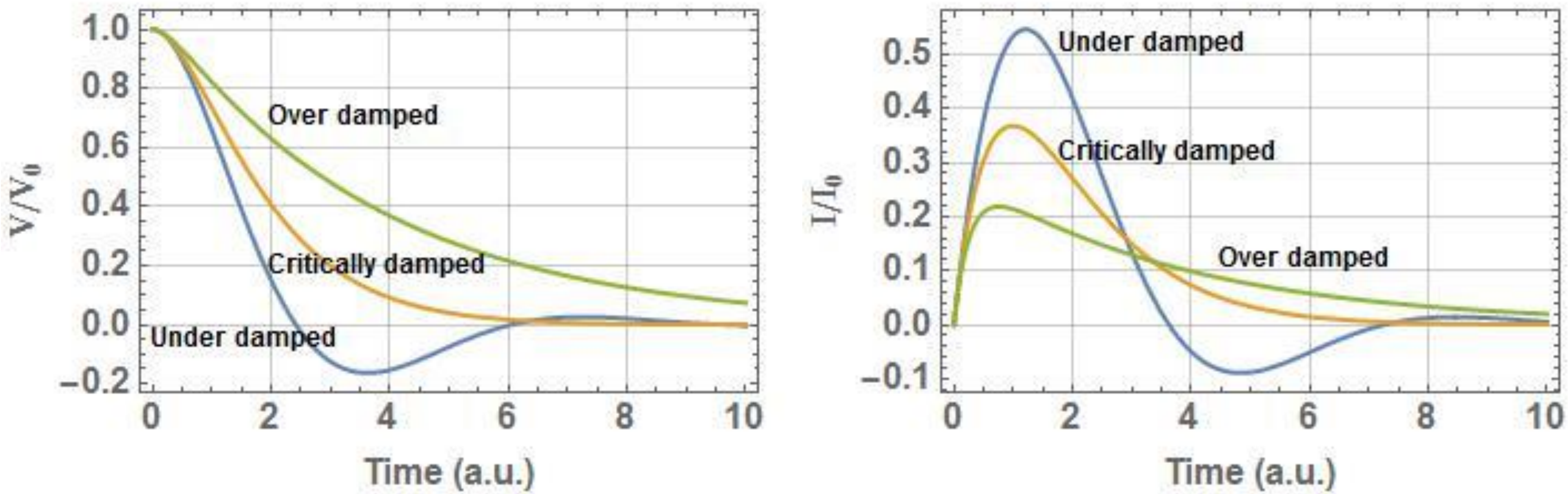
$$i = -C\frac{\mathrm{d}V_C}{\mathrm{dt}} = -C\left(\beta\exp\left(-\frac{R}{2L}t\right) - \frac{R}{2L}(V_o + \beta t)\exp\left(-\frac{R}{2L}t\right)\right)$$

$$i(0) = -C\left(\beta - \frac{R}{2L}V_o\right) = 0 \qquad \beta = \frac{R}{2L}V_o$$

$$V = V_o\left(1 + \frac{R}{2L}t\right)\exp\left(-\frac{R}{2L}t\right)$$

$$i = \frac{V_o}{L}t\exp\left(-\frac{R}{2L}t\right)$$

Varying R can move the discharge currents into different regime



$$R_{\text{cri}} = 2\sqrt{\frac{L}{C}}$$